FTD-MT-24-345-70

FOREIGN TECHNOLOGY DIVISION

FUNDAMENTALS OF DETAILED CALCULATIONS OF REINFORCED PLASTICS

(Chapter I)

by

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UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Foreign Technology Division Air Force Systems Command U. S. Air Force	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED
	2b. GROUP

3. REPORT TITLE

FUNDAMENTALS OF DETAILED CALCULATIONS OF REINFORCED PLASTICS (CHAPTER I)

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Translation

5. AUTHOR(S) (First name, middle initial, last name)

Tarnopol'skiy, Yu. M. and Roze, A. V.

6. REPORT DATE 19[illegible]	7a. TOTAL NO OF PAGES 67	7b. NO OF REFS
8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO.	FTD-MT-24-345-70	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d. DIA Task No. T70-01-03		

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Foreign Technology Division Wright-Patterson AFB, Ohio

13. ABSTRACT

Chapter I from a book on reinforced plastics deals with features of the structures and properties of fiber-reinforced materials. The materials are classified as to structure of the reinforcing fibers and subsequent reference to parts from fiber-reinforced plastics is in terms of the ideal model. Regular and random twisting of the reinforcing fibers, composite materials with small initial irregularities and preliminary twisting of the reinforcement make up the remaining general areas. Orig. art. has: 11 graphs, 12 tables, and 11 figures.

DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Fiber Composite						
Plastic						
Reinforced Plastic						

UNCLASSIFIED

FTD-MT-24-345-70

EDITED MACHINE TRANSLATION

FUNDAMENTALS OF DETAILED CALCULATIONS OF REINFORCED PLASTICS (Chapter I)

By: Yu. M. Tarnopol'skiy and A. V. Roze

English pages: 67

Source: Osobennosti Rascheta Detaley iz Armirovannykh Plastikov, Izd-vo Zinatne, Riga, 1969, pp. 13-65.

This document is a SYSTRAN machine aided translation post-edited for technical accuracy by: Joseph E. Pearson

UR/0000-69-000-000

FTD-MT-24-345-70 A Date 7 May 1971

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А. В. РОЗЕ

ОСОБЕННОСТИ РАСЧЕТА ДЕТАЛЕЙ ИЗ АРМИРОВАННЫХ ПЛАСТИКОВ

ИЗДАТЕЛЬСТВО «ЗИНАТНЕ»
РИГА 1969

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

FOLLOWING ARE THE CORRESPONDING RUSSIAN AND ENGLISH DESIGNATIONS OF THE TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}
rot	curl
lg	log

Translator's note: On several occasions, symbols found in formulae and calculations appear to have been rendered incorrectly in the original document. They will be shown exactly as they appear in the original.

CHAPTER I

STRUCTURAL CHARACTERISTICS AND PROPERTIES OF MATERIALS REINFORCED BY FIBERS

§ 1.1. Anisotropy

1.1.1. *Classification of materials reinforced by fibers.* The basis for classifying the materials of the pliable matrix-continuous reinforcing fibers type can be taken as the mutual arrangement (orientation) of the reinforcement. By this principle we can distinguish three basic groups of materials: unidirectional, also called fibrous, laminate and three dimensional-cross linked. If all the reinforcing elements are parallel, then it is customary to assume that composite substance has a *unidirectional*, or *fibrous*, arrangement; if the reinforcing elements are at an angle to one another in two or several parallel planes, then the arrangement is called laminate and, finally, if between the reinforcing layers there are cross connections, it is three dimensional-cross linked. The classification of materials reinforced by fibers is represented in Diagram 2.

Currently the production of reinforced materials uses unidirectional fibers (thread, coarse linen, braid) and fabrics of different interweaving both inside the ply (linen, serge, sateen, cord, etc.), and between plies (so-called "multi-ply fabrics of volume weaving"). When using nonfabric fiber as reinforcement, fibrous (if all fibers lie in one direction) or laminate (if in neighboring plies, fibers are at an angle to one another) products can be obtained. These are articles with stitched reinforcement, in which every ply is made up of parallel unidirectional fiber. Most frequently a mutually

orthogonal structure of fibers is used; moreover the ratio between longitudinal and transverse plies it can be different (1:2, 1:3, 1:5, etc.). Materials with 1:1 arrangement are called balanced (or of uniform strength).

When using fabrics of the usual weave products with a laminate arrangement are obtained, while different combinations are possible: the widths of the fabric can lie so that the direction of the warp in all plies coincides or between these directions of adjacent plies is formed a certain preassigned angle. For these materials the quantity of reinforcing strands in different directions is determined not only by the design of the fabric, but also by the preassigned scheme of piling the plies. Multiple fabrics are used for products with a three dimensional-cross linked structure, while existing designs of fabrics of volume weaving guarantee a transverse cross-linking only in the planes in which lie the fibers of the warp. For these materials the ratio between reinforcing fibers depends on the design of the fabric.

The analysis of microsections [101, 114, 209] of parts of unidirectional glass-fiber reinforced plastics allows considering that the fibers are located practically evenly along the cross section, while the geometry of their packing changes depending on the percent content of reinforcement in the material. In developing a model for determining the elastic constants of the composite materials it is necessary to rely on some diagram of fiber structure. Most frequently one relies on schematization of packing in the form of a quadratic (see, for example, [201, 281]), hexagonal [64, 253] or laminate [45, 173] structure of fibers.[1]

Detailed analysis of existing working methods of determining elastic constants (a detailed survey is contained in [191, 211]) indicated that for unidirectional materials the layout of the

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[1]Reinforcement by hollow fibers is examined in [65, 253].

arrangement of fibers has little effect on the properties in the direction of the reinforcement, but substantially affect the transverse characteristics. For example, the shear moduli of composite materials are a function not only of the shear modulus of the matrix, but also the parameter which describes the geometry and arrangement of the reinforcing fibers [252]. At the same time the majority of equations given in literature for determining elastic constants of the composite materials does not contain the parameters which characterize the structure of the reinforcing elements.

Only the values of Young's modulus in the direction of the reinforcement have been experimentally tested; however, for this characteristic all working methods give close values; the inaccuracy of these methods does not exceed 10%. Recently data appeared on the transverse characteristics (Fig. 1.1.1) [245], indicating a great variance. The guide for experimental appraisal apparently must be transversal moduli and shear moduli, as more sensitive to the packing of the reinforcement fibers.

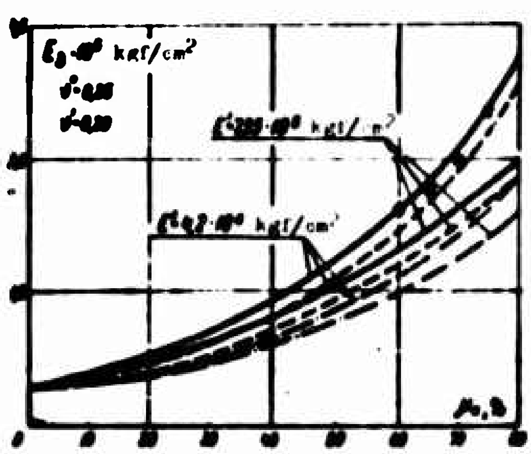

Fig. 1.1.1. Modulus of elasticity in the transverse direction depending on the volume content of fibers μ_x for unidirectional epoxy plastics, reinforced by fibers of boron and glass. —— according to [245]; --- according to [253]; -.- according to [267]. ν' and ν'' – Poisson coefficients of fiber and matrix. Experimental points: boron fiber – $E' = 4.2 \cdot 10^6$ kgf/cm^2; polymeric matrix – $E'' = 4.2 \cdot 10^4$ kgf/cm^2.

Because of a lack of vast experimental data today it is difficult to give preference to any method. During the experimental check of obtained equations all the necessary elastic constants will be found directly from experiments.

1.1.2. *Structural anistropy.* The examined composites are heterogeneous materials which possess structural anisotropy. Macrostructure characteristics allow in most cases maximum change to a quasi-homogeneous aeolotropic medium. This procedure becomes possible as a result of many reinforcing elements and the fact that the materials reinforced by the fibers possess a regulated heterogenity. For example, the characteristics of the structure of laminate materials are such that they can be considered as a discontinuous orderly medium composed of a great number of alternating plies of reinforcements and layers of polymeric binding, where "rigid" plies in the form of fabric, rovings or individual built-up fibers in the plies alternate regularly with "soft" interlayers of polymeric compound.[1] Multilayer substances can be composed of flat layers – rods, plates – or equidistant, for example, concentric layers, which possess the form of bodies of rotation – shells. Examples of multilayer models of oriented materials (plane and circular) are in Fig. 1.1.2.

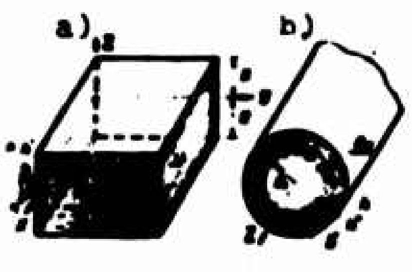

Fig. 1.1.2. Multilayer models: a) flat, b) circular. h' – depth layer of reinforcing fibers; h'' – depth of polymeric layer; h – depth of elementary layer, composed of reinforcements and polymeric binding.

[1]The reinforcing fibers as a rule can be considered as absolutely rigid in comparison with the pliable matrix. Thus, for glass fiber $E'/E'' \simeq 20$-50; for boron fiber $E'/E'' \simeq 100$-120 [260], where E'' – modulus of elasticity of the pliable matrix, E' – modulus of elasticity of the fibers. In a number of cases, as V. P. Nikolayev indicated [148], this can require the consideration of flexural effects in the rigid layer.

Introduction of the idea about a multilayer substance allows using parts from materials reinforced by fibers, the theory of reinforced substances of V. V. Bolotin, ideal parts and even with small initial imperfections in order to describe the stressed-deformed state. Later the technology of maximum change will be thoroughly shown in examples of a rod and an axisymmetrically loaded cylindrical shell. The meaning of a maximum change is that instead of the constructively aeolotropic discontinuous multilayer substance a continuous aeolotropic substance is introduced. In this case we can take into account the basic, determining features of the components of the material, make use of the apparatus of differential equations instead of a system of finite-difference equations for figuring stresses and strains, and then, if there is a necessity, return to the discontinuous substance.

The basis of the theory of reinforced substance is the macro-structural characteristics of the considered materials: rigidity of the fibers substantially exceeds the rigidity of pliable matrix; the depth of reinforcing elements and distance between neighboring elements are small in comparison with the characteristic dimensions of the bodies. These characteristics allow expressing the stressed-deformed state of a composite through the parameters which characterize the stressed-deformed state of the components. Subsequent realization of this means of describing the state of a reinforced body leads to a system of a very large number of equations. The obtained equations are differential in terms of coordinates (in the planes of reinforcement) and difference in terms of the indices attributed to the reinforcing elements.

The following fundamental step consists of using many reinforcing elements and the assumption that during the transition from one reinforcing element to another the appropriate functions are changed rather slowly. This allows approximating a finite, but very big number of functions of one or two coordinates (depending on the type

of the reinforcements) by several functions of three independent coordinates. The procedure makes it possible to obtain instead of the large number of differential-difference equations a system of several differential equations which describe the deformation of a certain given quasi-homogeneous medium. Thus it is possible to replace a heterogeneous material by a certain uniform aeolotropic material. The totality of procedures which lead to equations for an equivalent quasi-homogeneous aeolotropic medium is called by V. V. Bolotin the "principle of spreading."

The doubtless advantage of a theory of the reinforced media is the possibility of rejecting the idealized model of a material. This theory is easily extended to real materials, the reinforcing fibers of which have small initial distortions.[1] The fact that the shown approach does not superimpose any limitations on the dependence $\sigma \sim \varepsilon$ for components of a material is essential (the binding and reinforcement can possess arbitrary rheological features). For example, the behavior the bindings of the majority of glass-fiber reinforced plastics can be described with the help of the model of a "typical body" [200]. Extension of the theory of reinforced media to the case when a pliable matrix is a "typical body" is given in works of V. L. Blagonadezhin [35], Ye. N. Sinitsin [198], V. I. Fabrikant [246].

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[1]To evaluate the possibility of maximum change for materials with initial imperfections V. V. Bolotin [42] introduced into analysis three linear scales, differing in order of amount: $h \geq \lambda \geq \Lambda$, where $h \sim h', h''$. (h' – depth of reinforcing layers, h'' – depth of polymeric layer); λ – distance at which the functions describing the initial distortions of the layers change by a significant quantity. In turn, the characteristic wavelengths and correlation scales for the shown functions are small in comparison with the distances Λ at which the parameters of the layers can be considered constant, and the average stressed-deformed state as uniform. In this way, at level h the material is substantially heterogeneous, at level λ we apply maximum change to the quasi-homogeneous equivalent aeolotropic medium, at level Λ the material is stochastic [47].

Reinforcing elements can be one-dimensional (strands, rods) and two-dimensional (diaphragm, plates, shells, etc.). It is accepted that the deformations of the reinforcing elements are adequately described on the basis of the preconceived idea of undeformable normals, and all components of the stressed-deformed state in the binding layers, except the transverse shears and the elongation of the normals and corresponding stresses, are negligibly small. It is accepted that nonzero components of the stressed-deformed state are constant in depth. The difference in the Poisson coefficients is not taken account of (the high accuracy of the last assumption for oriented glass-fiber reinforced plastics has been proved in [126]), also not taken account of are the moment effects, whose role can show up only at a low content of reinforcements and at a very great difference between the moduli of elasticity of rigid and pliable layers.

The accuracy of maximum change, and consequently, of the theory of reinforced substances increases with the increase of the number of reinforcing elements in a material. Juxtaposition of the solution for lamellar (for example, by the method of A. R. Rzhanitsin [182]) and quasi-homogeneous substances makes it possible to show the number of layers at which is possible a maximum change to continuous medium. Research [36, 37, 186] indicated that at a number of plies $N \geq 10$ for the problems considered subsequently the error introduced by maximum change becomes insignificant. Hence it follows that for the majority of real designs from materials reinforced by fibers containing a considerably greater number of plies the replacement of a discontinuous substance by a quasi-homogeneous is entirely legitimate. This method is widely used in designs from glass-fiber reinforced plastics (see, for example, [127]).

After maximum change the materials reinforced by the fibers (fibrous, laminar three-dimensional-cross linked macrostructure) can be considered as transverse-isotropic or orthotropic bodies with rectangular or curvilinear orthotropy. With unidirectional

reinforcement articles possessing plane isotropy, perpendicular to the direction of reinforcement are obtained. Consequently, materials reinforced in one direction are transverse-isotropic materials. Strictly speaking, parts made from unidirectional semifinished fiber, even when it is stacked in one direction, do not possess transversal isotropy. The characteristics of the technology lead to a laminate, but fibrous arrangement because of the layers of polymeric binding between the plies of roving. Consequently, these products are orthotropic. A fibrous arrangement and the corresponding transversal isotropy are obtained only when parts are made by winding from elementary filaments. Transverse-isotropic features are possessed by materials with the so-called "star structure," whose plane of isotropy matches the plane of arrangement. With "star" arrangement the reinforcements lie in such a way that the angle between the direction of strands in adjacent plies is 60° (see, for example, [21]).

Reinforcement by roving in two mutually perpendicular directions (any practically usable relationships of longitudinal and transverse, but not more than 10:1), and also by fabrics of different weaving makes it possible to obtain articles with a lamellar structure from three reciprocally orthogonal planes of elastic symmetry. Plane parts made from these materials possess rectangular orthotropy, and products which possess the form of bodies of revolution and made by winding possess curvilinear orthotropy. Orthotropic bodies (true, with greater error) can also include products from materials with a three dimensional-cross linked structure.

1.1.3. *Substantial anisotropy*. Materials with anisotropy of any type – natural, structural, technological – depending on the relationships between their main technical constants are divided into materials with weakly expressed anisotropy and substantially anisotropic. The latter, following S. A. Ambartsumyan [5], include materials for which have ratios of the type

$$\frac{E_1}{G_{13}}, \frac{E_3}{G_{13}}, \frac{G_{12}}{G_{13}}, \frac{E_1}{E_3}, \text{ etc.,}$$

(the meaning of the designation is evident) are more than or equal to 5.[1] Obtained as a result of maximum change, the quasi-homogeneous material due to a pronounced distinction of the deformation and strength properties of components both in treatment and after the preparation of parts is substantially anisotropic. The fact is that for materials with fibrous and lamellar structures shear rigidity and strength, and also properties in directions perpendicular to the reinforcing fibers basically are determined by the pliable matrix, while rigidity and strength of the material in the direction of the reinforcement are determined by the properties of the rigid and strong reinforcing fibers.

The importance of anisotropy is most distinctly exhibited when elastic and strength properties in the direction of fibers are compared with the resistance to interlamination shear and extension-compression in the direction perpendicular to the plane of piling of the reinforcing fibers. In Table 1.1.1 are the most characteristic experimental data obtained during tests of reinforced plastics with different arrangement of the reinforcements (glass fiber E, S-994, boron fibers). In the state of processing, for example winding, the parameter of anisotropy $\beta_E = \sqrt{\frac{E_\theta}{E_r}}$, which is the ratio of the modulus of elasticity in the direction of the fibers $\bar{E}_\theta$ to the transverse modulus $\bar{E}_r$, for oriented glass-fiber reinforced plastics is 30-70. It is necessary to note that reinforced plastics possess substantial anisotropy not only of the elastic and strength properties, but also of some physical properties (shrinkage, radio transmissivity and others) and thermophysical properties (see, for example, [57, 58]).

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[1]For a one-dimensional problem S. G. Lekhnitskiy [123] proposes to estimate anisotropy with the help of the coefficients

$$s_{1,2} = -\frac{1}{2}\left(\frac{E_1}{G_{12}} - 2\nu_{12}\right) \pm \sqrt{\frac{1}{4}\left(\frac{E_1}{G_{12}} - 2\nu_{12}\right)^2 - \frac{E_1}{E_2}},$$

which are treated as the parameters which characterize the amount of the anisotropy of a material. For an isotropic material $s_{1,2} = -1$. Consequently, the more $|s|$ differs from one the more a material is anisotropic.

Table 1.1.1 [illegible]

Anisotropy of properties	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
E_x/G_{xz}	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Π_x/Π_{xz}	[illegible]	[illegible]	[illegible]	[illegible]		
E_x/E_z	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
Π_z^+/Π_z^-	10 [illegible]	[illegible]	[illegible]	~[illegible]	[illegible]	

Subsequently materials for which at least one of the moduli of shear G and strength with respect to the tangential stresses Π_τ is an order lower than Young's modulus E and the modulus of strength with respect to normal stresses Π_σ along the direction of reinforcement, we will call weakly shear-resisting, and materials in which $\Pi_x^{+(-)}$ and $\Pi_z^{+(-)}$ differ by an order badly compression-resisting with the load perpendicular to the reinforcing fibers, or transversally weak. Here and subsequently the sign (+) notes extension, and the sign (-) compression; the x and z for unidirectional materials indicate the direction of applied force relative to the reinforcing fibers.

Data presented in Table 1.1.1 indicate that materials reinforced by fibers inherit not only positive, but also negative properties of the components. Most frequently the products exhibit weak shear resistance. The introduction of three dimensional-cross linked reinforcement considerably improves resistance to interlamination shear. Table 1.1.2 compares the strength and elastic properties of two types of glass-fiber reinforced plastics on binding EDT-10 with

an approximately identical percent of reinforcement, but various types of weaving of the fabric (multilayer with different angle of inclination of the strands of the warp (for large section, Fig. 1.1.3) and the usual textiles of sateen weave). The structure of three-dimensional weaving is optimized by the angle of inclination of the strands in the warp. This allows minimizing (in comparison with the usual woven material) the fall of the modulus of elasticity on the warp E_x (for the indicated materials they are close) and considerably gains in the modulus of interlamination shear G_{xz}. The less twisted the fiber of materials, the higher the strength during compression in this direction. Transverse bonds considerably improve shear resistance of materials reinforced by fibers. However, data indicate not only a growth of the modulus of interlamination shear, but also the appearance of a new negative characteristic of three dimensional-cross linked materials. For many examined structures [102] compression strength sharply drops due to the fact that the fibers of the warp have strong initial distortions.

Table 1.1.2. Comparative properties of laminate and three dimensional-cross linked glass-fiber reinforced plastics.

Characteristic	Three dimensional-cross linked structure: angle of inclination of strand in warp					Laminate structure (VPS-7)
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
$E_x(\cdot 10^5)$, kgf/cm^2	3,25	2,50	2,25	1,30	1,05	3,00
$E_y(\cdot 10^5)$, kgf/cm^2	2,38	1,94	1,87	1,98	1,80	—
$G_{xz}(\cdot 10^5)$, kgf/cm^2	0,26	0,29	0,30	0,32	0,36	0,16
$\beta = \frac{E_x}{G_{xz}}$	12,5	8,6	7,5	4,1	2,9	19,0
ν_{xy}	0,122	0,124	0,135	0,176	0,188	0,114
ν_{yz}	0,170	0,161	0,155	0,126	0,115	0,210
Π_x^+, kgf/mm^2	44,4	38,0	35,0	14,1	13,8	47,0
Π_x^-, kgf/mm^2	29,4	26,2	22,6	11,7	11,2	32,5
Π_x^+/Π_x^-	1,50	1,45	1,55	1,20	1,23	1,45
Volume content of binding (EDT-10)	0,37	0,45	0,50	0,43	0,50	0,33

NOT REPRODUCIBLE

Fig. 1.1.3. Large section of a laminate glass fabric with three dimensional-cross linked structure (10x). ω_0 – angle of inclination of strands of warp.

Weak resistance to shear is most frequently exhibited, but not the sole unpleasant structural characteristic of reinforced plastics. In a series of designs operating under external or internal pressure under conditions of nonstationary temperature fields, especially with an increase of the thickness of the walls, one additional characteristic of materials reinforced by fibers is exhibited – weak extension and compression resistance under a load perpendicular to the fibers in comparison with strength and rigidity in the direction of reinforcement (see Table 1.1.1 and Fig. 1.1.4). The transversal weakness of unidirectional materials is explained by the fact that when the load is perpendicular to the reinforcing fibers resistance is determined by the polymeric matrix. As a result of the very considerable concentration of stresses along the contour of the reinforcing strands the unidirectional reinforcement actually is the source of the initiation of cracks [114, 259].

It is necessary to note that with an increase of strength and rigidity of strands, i.e., the betterment of properties of a material in the direction of reinforcement is connected with comparative intensification of the negative features of reinforced plastics – weak resistance to shear and to compression perpendicular to the fibers (see Fig. 1.1.4). This characteristic can impede the realization of increased strength and rigidity of a material in types where it is not possible to avoid dangerous (for reinforced plastics) types of loading.

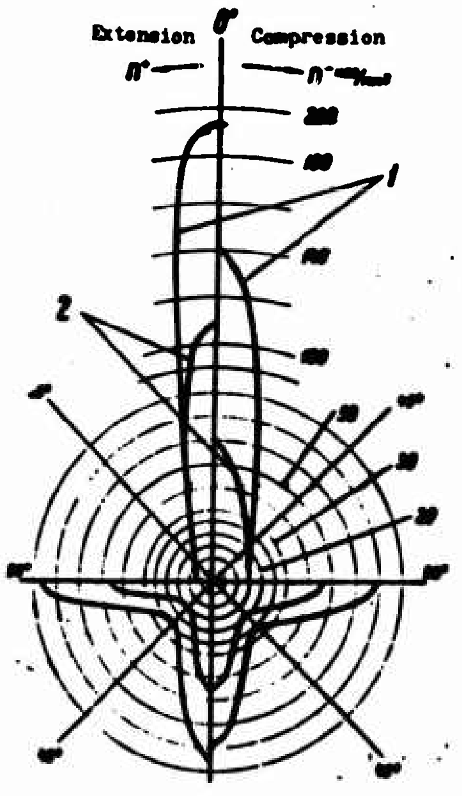

Fig. 1.1.4. Polar diagram of the strength of unidirectional and orthogonally reinforced glass-fiber reinforced plastics during extension and compression (kgf/mm^2) [267]. *1* – reinforcement – glass fiber S-994 (polymeric binding content 23%); *2* – reinforcement – *E*-glass fiber (binding content 33%).

1.1.4. *Geometric hypotheses; limits of applicability.* In engineering constructions the materials reinforced by fibers are used as a rule in the form of comparatively thin-walled rods, plates and of shells. Traditional during the calculation of such constructions is the application of hypotheses of flat sections (Bernoulli hypothesis) and undeformable normals (Kirchoff-Love hypothesis), i.e., neglecting shifts in the transverse planes and the compressibility of a normal. This simplifying assumption rarely introduces noticeable error during the calculation of thin-walled parts from isotropic or weakly aeolotropic materials, for which the moduli *E* and *G* are quantities of one order.

For composite materials it was necessary to take into account factors which ignore the usual engineering theories of streams, plates and shells. Hence as Yu. N. Rabotnov notes [176] this is a

revival of interest in the refinement of the hypothesis at the basis of basic equations and in ways of their possible correction.[1] Wear resistance to shear required refinement of the solutions obtained when using the hypothesis proposing infinite shear rigidity of a material: flat sections and straight normals. Transversal weakness in a number of problems led to the necessity of considering the compressibility of the normal. This caused a detailed analysis of the engineering solutions in which phenomena induced by the noted characteristics of these materials are not taken into account or are taken into account insufficiently.

For a number of problems, especially in loading by distributed slowly changing loads, the necessity of considering shears and compressibility of the normal can be determined on the basis of the concept about power error, introduced in [46]. Estimation consists in determining the contribution of the considered component (type $\varepsilon_x\sigma_x$, $\varepsilon_z\sigma_z$, $\gamma_{xz}\tau_{xz}$, etc.), to the amount of complete irrotational energy of a system. In this case it is assumed that for rigid layers the hypothesises of the usual theory of rods, plates and shells hold. In soft layers, on the contrary, there is a considerable departure from these hypothesises. If we consider individual layers [46], and the material as a whole (after maximum change) and use values for the appropriate elastic constants of real materials, then it is possible to obtain the desired criteria; this approach is widely used in subsequent chapters in the composition of basic equations. If we use the guides proposed by V. V. Bolotin [46], it is possible to write, retaining basically its designations:

[1]The hypothesis of undeformable normals, while geometrically formulated, has a distinctly expressed physicomechanical content [11]. Its accuracy in the geometric sense does not reflect the entire complexity of the picture connected with anisotropy and heterogeneity. Anisotropy of materials reinforced by fibers as a rule is so substantial that it forces one to introduce a correction not only to the formulation of the problems, but also to the customary definitions (thin shell, long shell, etc.), [5, 89].

$$
\begin{aligned}
\sigma_x \varepsilon_x &\sim \varphi_x \bar{E} \bar{\varepsilon}^2, \\
\sigma_z \varepsilon_z &\sim \frac{\eta^4}{\varphi_z} \bar{E} \bar{\varepsilon}^2, \\
\tau_{xz} \gamma_{xz} &\sim \frac{\eta^2}{\psi_{xz}} \bar{E} \bar{\varepsilon}^2,
\end{aligned}
\qquad (1.1.1)
$$

where $\bar{E}$, $\bar{\varepsilon}$ – characteristic moduli of elasticity and deformation of a material; ϕ_x, ϕ_z, ψ_{xz} – parameters which connect characteristic stresses and deformations with the other characteristics:

$$E_x = \varphi_x \bar{E}, \quad E_z = \varphi_z \bar{E}, \quad G_{xz} = \psi_{xz} \bar{E};$$

$\eta = \frac{h}{l}$ – certain small parameter which is determined by the relationship to the characteristic dimensions of the construction.

For problems of the bending of rods and plates small parameter η has the order of the ratio of the height of a rod or a plate to the span $\eta = \frac{H}{l}$. The ratio to parameters $\frac{\phi_x}{\psi_{xz}}$ for glass-fiber reinforced plastics can reach 50-60, and for boron plastics even 100. Hence it is apparent that for the majority of structural uses of rods and plates from materials reinforced by fibers, the contributions from normal and shear components to the irrotational resilience are quantities of one order. Using the same considerations it is possible to show that for oriented plastics in extension and bending deformations in the direction of the z-axis ($\varepsilon_z \simeq 0$; $\sigma_z \simeq 0$) can be neglected. For these materials ϕ_z and ψ_{xz} are close to one another, since E_z and G_{xz} are close. Consequently, for these structures a correction from transversal stresses and deformations is η^2 times less than the correction introduced by the consideration of transverse shears.[1]

[1]An analogous result for multilayer plates has been obtained by R. M. Rappoport [179], who indicated acceptability for considered media of the principle of partial consolidation ($\varepsilon_z = 0$), since the effect of σ_z on deformation ε_x and ε_y is negligible.

Consideration of the compressibility of the normal can be required only for materials with a very pliable matrix and fibers of great rigidity.

In the delivery state for unidirectional and woven materials $\phi_x \gg \phi_z$; the shown parameters can differ by more than three orders. In this instance, for example in winding thick-walled rings, the contribution from the compressibility of the normal becomes commensurable with the contribution to the complete potential energy of a system from normal stresses in the direction of reinforcement. The comparative contribution from the compressibility of the normal

$$\frac{\sigma_z \varepsilon_z}{\sigma_x \varepsilon_x} \sim \frac{\eta^4}{\varphi_x \varphi_z}.$$

In this instance $\eta = \frac{h}{R}$ is the ratio of the thickness of the ply of wound material h to the radius of the part being wound R, consequently, for many real processes of winding (depending on anisotrop and geometry) consideration of the compressibility of the material while winding can be required.

For oriented glass-fiber reinforced plastics transverse shifts must be taken into account in problems of bending [219]. Consideration of the compressibility of the normal is necessary during the analysis of the mechanics of the processes of winding these materials, even when winding rings which by tradition are accepted as thin-walled.

§ 1.2. Idealized Model of a Material and Technological Defects

1.2.1. *The idealized model.* In the previous section in the description of structurally anisotropic materials reinforced by continuous fibers, depending on the scheme of interweaving of the reinforcement (inside an elementary layer or between layers) three

basic types of macrostructures were distinguished: fibrous, lamellar and three dimensional-cross linked ("volume"). The basis of working methods for designs from all these materials is an idealized model composed of the rectilinear reinforcing elements, orderly arranged in a pliable polymeric matrix. It has been accepted that the reinforcing fibers and polymeric matrix are distorted together. In actuality the macrostructure of all considered types of materials is far from this model. In real parts from materials reinforced by fibers (for example, from oriented glass-fiber reinforced plastics), it is practically impossible, at least at the present level of technology, to avoid technological micro- and macrodefects [114, 72]. Primary defects are distortion of the reinforcement, breakdown of the solidity of composition (both along the edges of the strands, and the polymeric matrix itself), the presence of microcracks in the reinforcing fibers and in the polymeric matrix, deviation from a uniform arrangement of strands over the cross section.

The idea that the materials reinforced by fibers at all stages of loading act as a continuous monolithic material was subjected to a thorough experimental and theoretical check because the experience of using parts from glass-fiber reinforced plastics and the results of a number of investigations disprove the assumption about the absolute solidity of a material at all stages of loading. Analysis of numerous experimental data and a theoretical description on the base a model of Outwater done in [212], made it possible to establish that for the basic volume of a material the preconceived idea of continuity is retained practically up to failure (the faulted volume as a rule does not exceed 2%). In creating methods of strength and rigidity calculation this allows considering reinforced plastics as continuous at all stages of loading. An exception is parts to which are presented requirements of air-tightness. In this instance the stress at which crack formation starts in a composite material should be the working stress.

Deviation of the reinforcing strands from straightness is one of the most frequent defects of the macrostructures of reinforced

plastics. The presence of distortions of reinforcing strands of all types of reinforced plastics is one of the reasons that the deformation and strength properties of these materials in parts are considerably different from characteristics obtained in samples. Still more different are the properties of parts from the characteristics calculated by different means for a material reinforced by ideally straight fibers.[1] The twisting of reinforcing fibers represent a specific danger for designs which operate on stability, since the presence of layers with reduced properties leads to local drops in rigidity. This causes a substantial drop in critical loads.

The moduli of elasticity in the direction of fibers, found on the basis of a large base of measurements, as a result of technological twisting of the fibers considerably differ from their local values. As an illustration Table 1.2.1 gives nimerical values of the modulus of elasticity E_x obtained during testing samples from AG-4S material (averaging on a 100 mm base of measurement), and also the results of the measurement of local values of the modulus of elasticity in places with maximum distortion of fibers, with loose twisting of fibers and without apparent deviations from straightness (measurement was made by strain gauges with a base of 20 mm).

From presented data (Table 1.2.1; the number of examples can be continued) it is evident that the effect of loose technological twisting can be very considerable. The sensitivity of reinforced plastics even to loose twisting of fibers is a consequence of the importance of anisotropy of the properties of the materials of the type of the pliable matrixcontinuous reinforcing fibers and primarily their weak resistance to shifting. This leads to the necessity of studying reinforced media allowing for small initial inaccuracies. Let us examine first the reasons which cause the twisting of reinforcing fibers, and classify these twistings.

---

[1]The twisting of strands also has substantial effect on the physical features of a material. In [70] the effect of the twisting of fibers on the coefficient of thermal conductivity of glass-fiber reinforced plastics is examined.

Table 1.2.1. Change in modulus of elasticity along a sample.*

Modulus of elasticity ($\cdot 10^5$), kgf/cm²	Sections along a sample			Averging on a base of 100 mm	After [253]
	with maximum twisting of fibers	with loose twisting of fibers	with straight fibers		
$E_{x\,cp}$	2,83	3,38	3,53	3,31	4,20**
$E_{x\,min}$	2,75	3,38	3,12	3,04	—
$E_{x\,max}$	2,93	3,44	3,80	3,45	—

*The presented data were obtained during tests of seven specimens, cut out from one plate along the long side. Places with twisting were selected in the plane perpendicular to the plane of the die. Plates (600 x 125 x 18 mm) were made by the usual technology and originally were intended for other purposes, so that the twisting of fibers in them was not intentional.

**With a volume percent content of glass fiber in the material ~60%.

1.2.2. *Orderly and random twisting of fibers.* The twisting of filaments of materials reinforced by fibers can have an orderly or random character. Interweaving in the interior of the elementary layer (in laminate woven materials) or between layers (in three-dimensional woven materials with cross-links between layers) give materials with preassigned orderly twisting of the reinforcing fibers. In laminate materials the twisting depends on the arrangement of the reinforcing fabric (various types of fiber weave being used to create laminate glass fabric are indicated, for example, in [147]; as a rule, fibers of the warp and woof distort). In three-dimensional fabrics the amount of twisting is determined by the pattern of cross linking, while in existing fabrics the warp fibers distort on a specific pattern; in the direction of the woof the fibers have no apparent twisting.

In making articles from woven materials additional twisting of the reinforcing fibers can develop. They are an unavoidable consequence of the existing technology of processing reinforced plastics into articles. The amount and character of the appearing

twisting are affected both by the means of processing, and by the accompanying formation of thermal and chemical shrinkage of the material [227]. Especially apparent is twisting of fibers of articles made by contact formation and pressing in closed molds. Technological twisting can be observed also in coiling (especially articles of large diameter) with low forces of tension and subsequent molding (for example, with the help of a "vacuum pocket") as well as with other widely used means of processing materials reinforced by fibers. Shrinking during polymerization intensifies the twisting of fibers of woven materials and, if this is not prevented by the force of the preliminary tightening it changes the picture of twisting of the fibers preassigned by the arrangement of **weaving.**

In this way, in woven materials twisting can be orderly (preset by the arrangement) and random (conditioned by formation). In laminate woven materials twisting, both orderly and random, as a rule is loose. The change in percent content of binding proves to have an insignificant effect on the curvature of the strands, and changes basically the distance between the plies. In three dimensional-cross linked materials orderly twisting of the fibers is preset by the warp. The amount of twisting depends not only upon the pattern of weaving, but also upon the percent content of binding. An increase of the percent content of binding increases the amplitude, which characterizes the amount of twisting of strands.

The technological features of processing materials reinforced by fibers lead to the fact that twisting of the reinforcement is observed not only in woven materials, in which it is unavoidable, but also in unwoven. The reason for this has been established with the aid of optic polarization method of Matting and Hafercamp [299] is the shrinkage processes during the polymerization of the binding. The contraction stresses lead to random twisting of fibers (Fig. 1.2.1), the amount of which depends on the means of formation (presence and amount of preliminary tightening of reinforcement and the properties of the polymeric matrix. For the last reason the

average values of the moduli of elasticity and strength during elongation in the direction reinforcement of unidirectional materials (with a fibrous arrangement) on a binding with low contraction is higher, but the variance of characteristics lower than in materials whose bindings have great contraction (Table 1.2.2).[1]

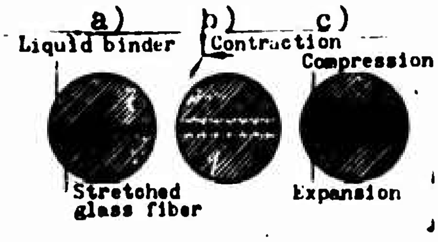

Fig. 1.2.1. Twisting of fibers in the process of polymerization (schematic representation). a) initial state; b) polymerization; c) final state.

Table 1.2.2. Properties in the direction of reinforcement of unidirectional glass-fiber reinforced plastics on different bindings.*

Modulus of elasticity and strength in the direction of reinforcement	Material with much contraction (27-63 C)	Material with small contraction (P 2-1)	For ideally straight fibers
$E_{x\,cp}(\cdot 10^5)$, kgf/cm²	4,2	4,75	5,00
$\frac{E_{x\,min}}{E_{x\,max}}$	$\frac{4,0}{5,2}$	$\frac{4,65}{4,85}$	—
v_{Ex}, %	8,2	4,5	—
Π_x^+, kgf/mm²	89	97	—
$\frac{\Pi_{x\,min}}{\Pi_{x\,max}}$	$\frac{84,0}{102}$	$\frac{95,5}{100}$	—
v_{Π_x}, %	9,9	3,3	—

*The reinforcing fiber content in both materials is approximately equal (the coefficient of reinforcement $\chi=0.7$). For ideally straight fibers $\bar{E}_1$ is calculated by addition; v – the variation factor. Samples were made in the same mold; distinctions consisted only in the amount of binding contraction.

[1]Strength and deformation chracteristics of reinforced plastics in many respects are determined by the parameters of formation: pressure, temperature and formation time. The effect of these factors has been examined comprehensively [208]. The data in Table 1.2.2, in subsequent tables and on graphs have been obtained in samples made in optimum conditions.

The deflection in the reinforcing fibers from straightness for nonwoven materials has a random character. During the preparation of constructions from unidirectional materials as a rule a small twisting of the reinforcement takes place. They can be the consequence of nonuniform arrangement of the reinforcing fibers in the mold, a drop of the tension force below critical during winding or technological (chemical and thermal) contraction of the binding. Especially apparent is the twisting of fibers of materials reinforced by roving in two perpendicular directions (Fib. 1.2.2). Sometimes we find very substantial local twisting of fibers (Fig. 1.2.3) of either the fatric induced, for example, by having in the mold sheets or strips of material longer than the mold. The chance for considerable local twisting of glass fibers increases with an increase in the over-all dimensions of the parts, causing additional difficulties connected with the arrangement and attitude of the reinforcing fibers in the mold. Diagram 3 indicates reasons for the initiation of twisting of the fibers of reinforced plastics, shows the nature of the twisting (orderly or random), amount and role of the tension of reinforcements in the formation process.

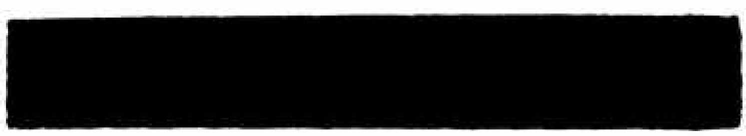

Fig. 1.2.2. Twisting of the layers of plates from AG-4S material reinforced in two perpendicular directions (arrangement 1:1, macrograph, mag. 2x).

Constructions from composite materials, including the rods, plate, thick-walled rings considered in the book, depending on the accepted means of formation can be made without tension or under conditions when the reinforcements are preliminarily stressed. The

tension of the reinforcements in the process of formation allows removing or, at least, diminishing the danger of twisting of the reinforcing fibers. Naturally, complete straightening of the fibers can be achieved only for unwoven materials; for woven (laminate and three dimensional-cross linked) the introduction of preliminary tension of the fibers leads to the decrease in twisting of the reinforcements in the direction of tension. Study of the macrostructure of woven materials (see § 1.4) testifies to the fact that tension decreases the amount of the orderly twisting of fibers preset by the pattern of weaving, and removes the effect of contraction.[1]

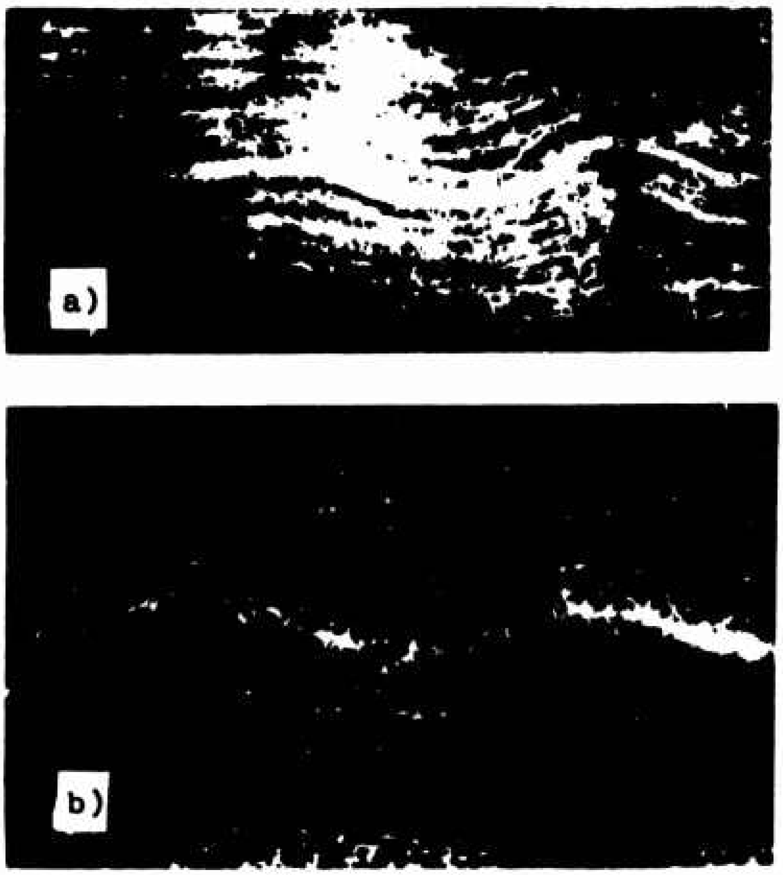

Fig. 1.2.3. Macrographs of local twisting of strands (mag. 2x). a) plate from a unidirectional material (AG-4S); b) thick-walled ring from woven material (VPS-7) (mag. 3x).

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[1]The force of tension leads to ellipsoidality of the reinforcing fibers. As notes G. D. Andriyevskaya [19], already in the process of weaving reinforcing fabrics the circular form of the cross section of the fiber strands becomes elliptic.

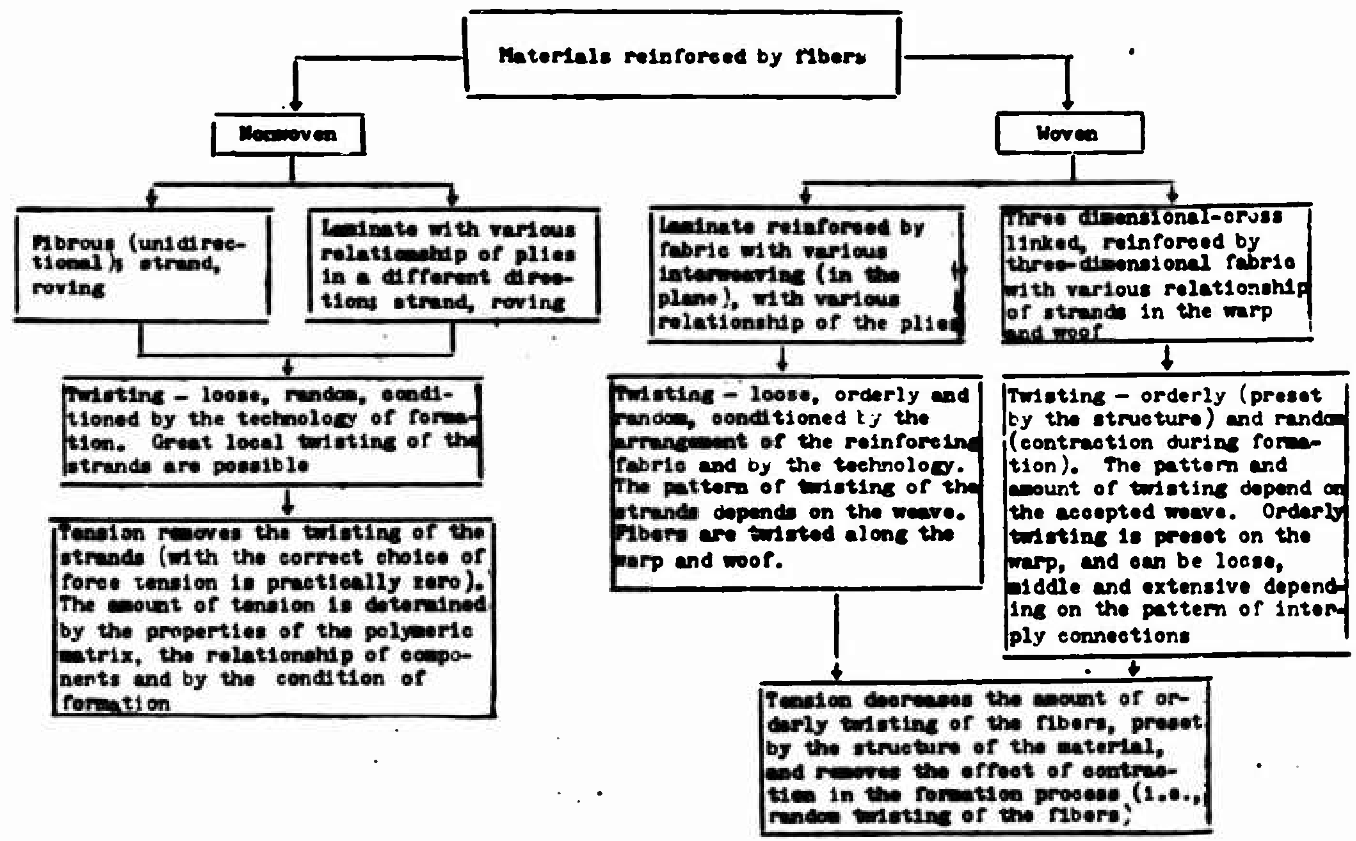

Diagram 3. Classification of materials by the structure of the weave.

1.2.3. *Deviations from the idealized model.* The twisting of the reinforcing fibers, both preset by the structure of weaving of the fabric, and random twisting, are the unavoidable consequence of the existing technology of the preparation of structurally aeolotropic materials and articles from them. The tension of the reinforcements does not remove completely the danger of twisting the reinforcing fibers; in most cases it only allows approaching the idealized model of the material. There is a persistent necessity of estimating the effect of deviation from the ideal model and estimating the nearness in elastic and strength properties of all three basic types of the considered composite materials. To estimate departure from the ideal model of a material and the role of subsequent tension of the fibers the theory of reinforcement of substances with small initial inaccuracies has been used, on the basis of which the effect of tension of the reinforcements on streugth and deformation of oriented materials has been examined.[1]

We will use for the ideal model the indexes 1, 2, 3 (for example, E_1, E_2, G_{13}, etc.), but for real materials allowing for of the twisting of the fibers – x, y, z (for example E_x, E_z, G_{xz}, etc.). For further discussion it is essential that the effect of twisting the reinforcements (this will be indicated in § 1.3) is exhibited mainly in the direction of the fibers and has little effect on resistance, which is determined basically by the polymeric matrix. Therefore consideration of various secondary effects (for example, the arrangement of the fibers) during the analytic determination of elastic characteristics in the direction of reinforcement is not advisable. In such cases, as notes H. N. Rabotnov, "the consideration of subtle secondary effects hardly has meaning if it is necessary to use experimental data obtained in samples, and there is no confidence that the material of the article possesses the same characteristics, and moreover, is sufficiently uniform" [174]. At the same time a thoroughness in creation of models for determination

[1]A survey of a few theoretical and experimental data about the effect of tension of the reinforcements on the properties of glass-fiber reinforced plastics is given in the works of G. G. Portnov [167] and I. G. Zhugun [102].

of deformation characteristics, determined by the polymeric matrix and arrangement of the strands is necessary. This is especially important, allowing for the inelastic properties of the binder. For transverse characteristics the twisting of fibers is a secondary effect and need not be considered.

1.2.4. *The setup and technology of the preparation of samples.* Experimental research on the deviation of a material from an ideal model and the role of subsequent tension of the fibers has required the development of a special technology and equipment with the consideration of the specific property of materials with various patterns of reinforcement. Research was carried out on plane and circular samples. Plane samples were made in the form of strips or plates, which then were cut into samples of preassigned sizes.

The overall view of the device for preparation of plane samples (strips) is in Fig. 1.2.4. The device consists of a mold (Fig. 1.2.4a) and a special attachment which turns the frame and the tightening device (Fig. 1.2.4b, c). In order to facilitate the removal of the sample, the matrix of the mold has only two side walls (Fig. 1.2.4d). To provide uniform heating through the entire length of the sample the electrical heating was conducted by sections having separate temperature control. Temperature was selected for each section on the basis of readings from eight thermoelectric couples installed in the mold. The mold was mounted on a P-63 hydraulic press. The device is intended for the preparation of samples of three types: with preset orderly twisting of strands, with reinforcement straightened before pressing but unstrained and with preliminarily stressed reinforcement. In the preparation of samples in the form of bands a unidirectional strip of the being examined materials (width 20 mm) was wound on a special device with tension or without it. The preset twisting was induced when necessary with the help of special bushings. The appliance turned on force-measuring element, which allows measuring the force of tension at all stages of preparation

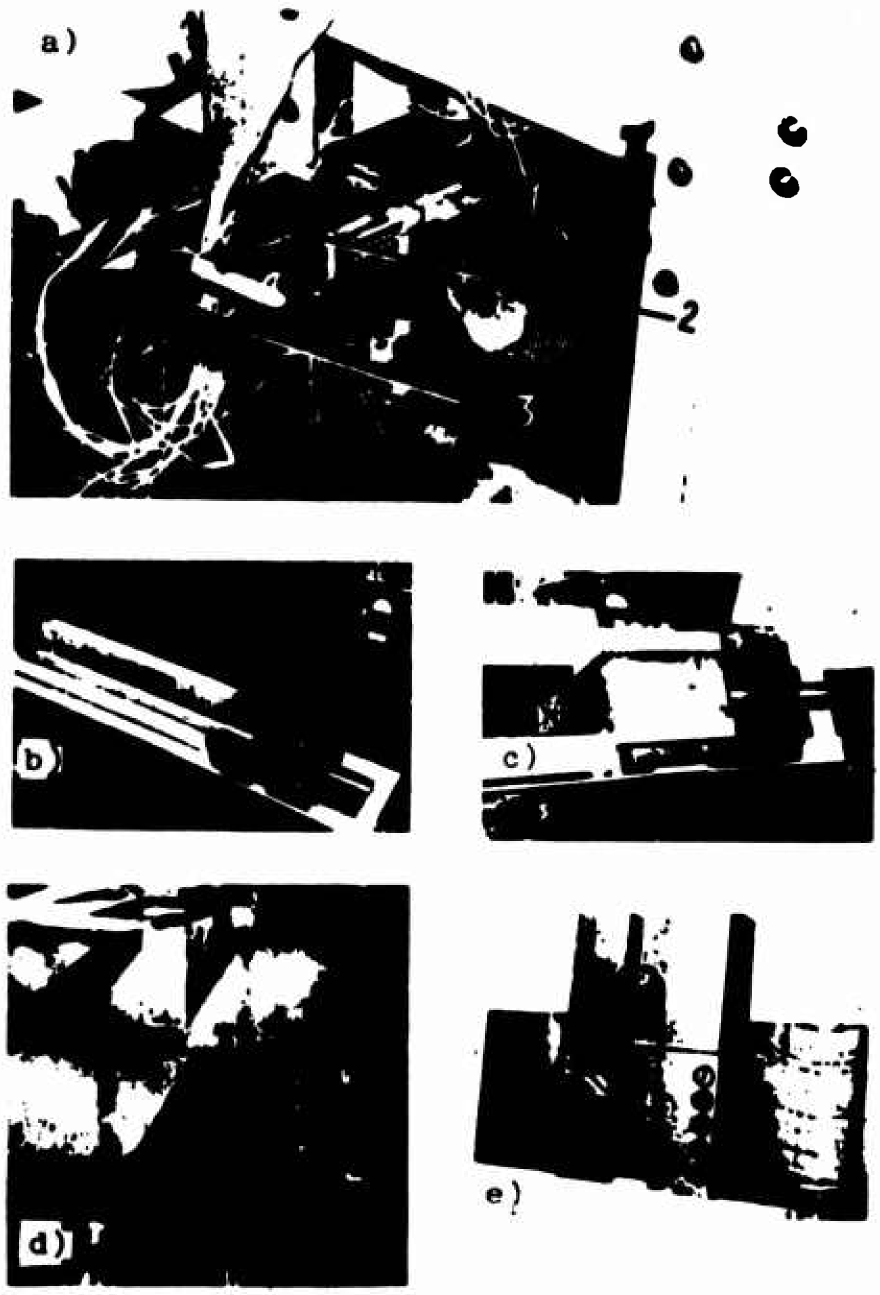

Fig. 1.2.4. An experimental device for the preparation of samples with preset twisting of the strands and preliminary tightening of the reinforcement. a) overall view of mold *1* and tightening equipment *2*; b) detachable frame *3* for winding strip samples; c) tightening equipment with spacer linings for preparation of strip samples; d) matrix and mold punch; e) spacer linings.

of the sample . The uniformity of the tension in all plies was provided by s spacer bushings inserted into the bar (Fig. 1.2.4e). Each loop of the strip was wound directly to the bushing, and not one to another, which avoided changing the force of tension in the loops over the height of the sample as a result of strain of the underlying layers of the strip [213] at the isochronous application of strentching force to all loops (bundle) and prevented the twisting of ready samples. Thus it was managed to master the technology of obtaining plane samples with approximately uniform distribution of tension in the depth. The strip was tightened with a screw passing through the bar and resting in the force-measuring element. The force-measuring element was a steel arm with glued-on strain gauges, which allowed judging the total tension of the fibers.

The plates were made from woven materials (laminate and three dimensional-cross linked) on a special setup (Fig. 1.2.5), having an attachment for tightening, which made it possible to make plates of three types: with straightened, but unstrained reinforcement, with tension on the basis and woof and with tension in one of these directions. In other respects the technology of preparation was the same as during the preparation of band samples. The basic difficulty was fixing the ends of the reinforcing fabric [101].

Circular samples were made by winding (Fig. 1.2.6) with constant tension N_0 onto special mandrels (Fig. 1.2.7a-c). The mandrels, shown in Fig. 1.2.7a and b, were equipped with heat-resistant strain gauges, which allowed measuring the total tension on the mandrel at all stages of the preparation of circular samples. The total drop of tension in the loops was estimated. Winding into a track (Fig. 1.2.7b and c) prevents the material from spreading apart during the winding of rings with a large number of loops with high tension. The technology of winding rings with an investigation of individual stages is presented in [59, 62].

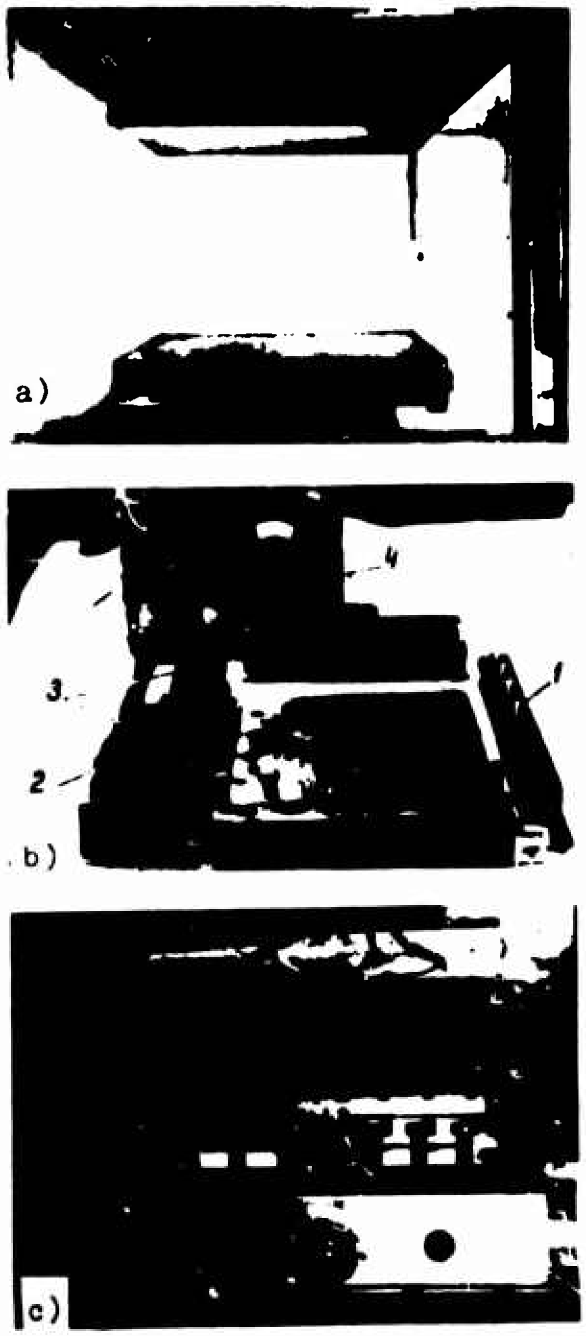

Fig. 1.2.5. Device for the preparation of plates. a) mold; b) tension device: *1* – frame, *2* – force-measuring elements; *3* – tightening device, *4* – static deformation gauge ISD-2; c) mold and tension device (in the collection).

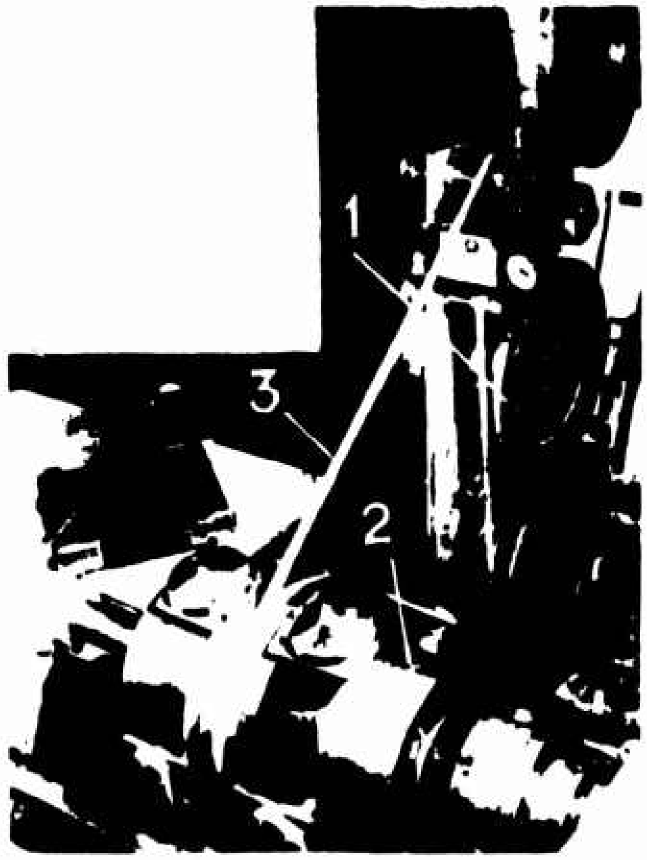

Fig. 1.2.6. Overall view of device for winding circular samples: *1* – winding equipment, *2* – force-measuring mandrel; *3* – material being wound.

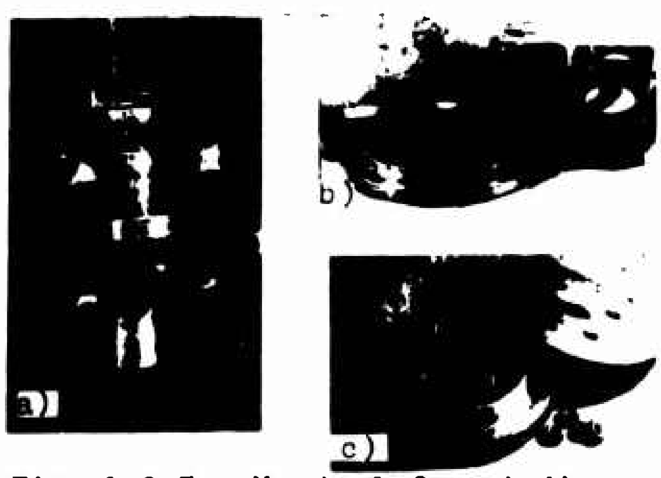

Fig. 1.2.7. Mandrel for winding; a) force-measuring mandrel (without limitations on width); b) force-measuring mandrel for winding into track; c) for winding into track.

During the preparation of preliminarily stressed plane and circular samples the preset tension was changed within limits of 0.1-0.6 Π_M (Π_M – strength of a band of the starting material in the winding state). When the preparation of plates the maximum tension on the warp and woof did not exceed 0.25 Π_M. The described devices allowed simultaneously making two samples 400 mm long or one plate 250 × 250 mm^2.

A feature of the preparation of preliminarily stressed samples of reinforced plastics is that as a result of the presence of polymeric binding substantial distinctions are allowed between the actually retained tension $N_{эф}$ and the initially preset N_0. Therefore during the preparation of plane samples the initially preset tension of the reinforcing fibers was checked during the whole technological process. It was shown that for all the examined materials the formation process is accompanied by a drop of the initial preset tension. Composite data for all three types of materials are presented in Fig. 1.2.8. Each point on the graph represents on the average 12 samples; the variation factor v_T = 3.2-10.7%. The experimental points were obtained during the preparation of ten-layered samples. As it appears from the presented data, in ready products only one third of the initially preset tension can be kept.[1] Tension in two mutually perpendicular directions does not exclude contraction. Experiments indicated that during the preparation of plates the contraction in magnitude is close to that obtained on band samples.

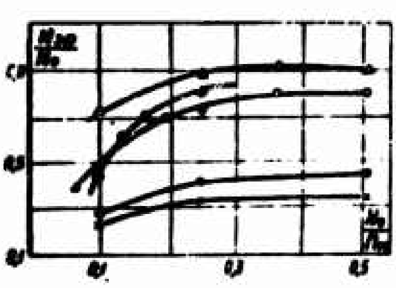

Fig. 1.2.8. Actual tension of reinforcing fibers. O – unidirectional material AG-4S; × – woven material SKT-11; Δ – three dimensional-cross linked material (tension along woof); ● – three dimensional-cross linked material (tension along the warp); ▲ – three dimensional-cross linked material (isochronous tension on warp and woof.

[1]In samples made from materials whose matrix possesses creep, during storage there is a further drop of tension. For oriented glass-fiber reinforced plastics this question has been studied by A. M. Skudraya [200]. The analysis of the reasons for contraction is in [72, 234].

In this way, consideration of the drop in tension during studies on strength and rigidity of preliminarily stressed samples is necessary in many practically important cases. Subsequently all experimental data, except that obtained for circular samples, are presented depending on the actual force of tension.

§ 1.3. Composite Materials with Small Initial Irregularities

1.3.1. *Stochastically reinforced substances.* A theory of reinforced laminate substances with small random initial irregularities has been developed by V. V. Bolotin and his colleagues (see, for example, [44, 47] and others). These works consider elastic and viscoelastic environments, reinforced by slightly twisted elastic layers. On the assumption that the functions which describe the initial twisting of the ply form a random field, formulas were obtained for the determination of statistical stresses, deformations and movements in the reinforced substance. This opens a fundamental possibility of estimating the effect of the twisting of fiber on all components of the compliance tensor of orthotropic materials.[1] Although the obtained expressions are lengthy, they can be considerably simplified for a number of important applications. For example, the expression for the modulus of elasticity in the direction of the fibers E_x has the form [47]:

$$E_x = \frac{\chi E''}{1+\frac{\chi}{s^*}E''\int\limits_0^\infty\int\limits_0^\infty\int\limits_0^\infty k_x^2\Phi_{ww}(k_x,k_y,k_z)\,dk_x dk_y dk_z}; \tag{1.3.1}$$

$\Phi_{ww}(k_x,k_y,k_z)$ - spectral density (k_x, k_y, k_z - wave numbers); parameter χ has the meaning of the volume coefficient of reinforcement;

$s^* = \frac{G''}{1-\chi}$ - minimum critical stress of a local protrusion. Within the framework of the considered theory the expression for s^* is identified with the shear modulus of the reinforced material.

---

[1]Of problems considered in the book those of greatest interest are the elastic constants necessary to investigate curvature and stability of rods and plates (without a hypothesis of nondeformed normals), and also the behavior of thick-walled rings under pressure.

With small inaccuracies

$$\int_0^\infty\int_0^\infty\int_0^\infty k_z^2\Phi_{w_z}(k_x, k_y, k_z)dk_xdk_ydk_z=\psi_\sim^2$$

is the mean quadratic value of the angle which a fiber makes with the direction of reinforcement.

By the conditions of the problem, $\psi_\sim^2 \ll 1$, therefore it is possible to obtain a simple formula for estimating the effect of small initial twisting of layers on the apparent modulus of elasticity during elongation E_x:

$$E_x=\frac{E_1}{1+\frac{E_1}{G_{13}}\psi_\sim^2}, \tag{1.3.2}$$

determined as the ratio between average stresses and deformations; namely, these deformations are measured on a preassigned measuring base.

The shear modulus in the plane of reinforcement G_{xy} is described by the expression

$$G_{xy}=\frac{\chi G'}{1+2\chi G''\int_0^\infty\int_0^\infty\int_0^\infty k_xk_y\frac{\partial F_{xy}(k_x, k_y, k_z)}{\partial\tau_{xy}}\Phi_{w_z}(k_x, k_y, k_z)dk_xdk_ydk_z}. \tag{1.3.3}$$

Here $F_{xy}(k_x, k_y, k_z)$ - transfer function; at $\sigma_x=\sigma_y=0$

$$F_{xy}(k_x, k_y, k_z)=$$

$$=\frac{2k_xk_y\tau_{xy}}{\frac{\chi E'}{1-\nu^2}(k_x^2+k_y^2)+\frac{G''}{1-\chi}(k_x^2+k_y^2)+\frac{E'}{1-\chi}k_z^2+2k_xk_y\tau_{xy}};$$

σ_x, σ_y, σ_z - values of nominal ("smeared out" with respect to the whole volume) stresses in the reinforced material; E', G'', E'' - moduli of elasticity of rigid and pliable layers.

At $k_x = k_y$ and with earlier assumptions the effect of the twisting of fibers on G_{xy} is described by an equation arranged as (1.3.2):

$$G_{xy}=\frac{E_1}{1+\frac{E_1}{G_{13}(1+\nu_{12})}\psi^2_{\sim}}. \qquad (1.3.4$$

1.3.2. *Estimation of the possibility of straightening of twisting strands during loading.* To answer this question it is necessary to evaluate the effect of amount of acting stress on the modulus of elasticity in the direction of the fibers, i.e., on the form of the relationship $\sigma\sim\varepsilon$ for the materials reinforced by slightly curved fibers. It is easy to assume that if during elongation (or contraction) there is a straightening (or, conversely, additional twisting) of the fibers, this leads to a change in $E_x^{+(-)}$ depending on the level of acting stress $\bar{\sigma}$ (Fig. 1.3.1). From the theory of reinforced substances it follows that at $|\bar{\sigma}| \ll s^*$ [42] the modulus of elasticity in the direction of the fibers does not depend on acting stress $\bar{\sigma}$. If we take into account (this was indicated earlier) that the minimum critical stress of local protrusion has the order of the shear modulus of the matrix, then to answer the posed question it is necessary to contrast the shown quantities. In existing reinforced plastics strength during elongation and especially during contraction is an order less than the modulus of the elasticity of the binding. Therefore for these materials, especially in the area of moderate stresses, the deviation of the dependence from a straight line is practically undetectable. This is well confirmed by numerous experimental data obtained during

tests of oriented glass-fiber reinforced plastics with straightened fibers and in samples with twisting of the strands [102, 215].

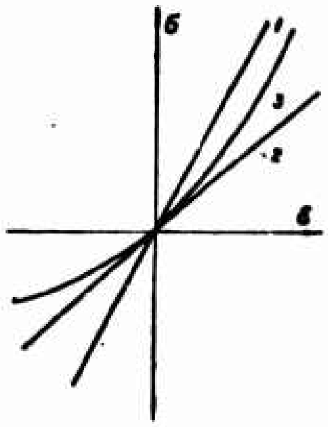

Fig. 1.3.1. Relationship between stresses and deformations: 1 - ideal model; 2 - twisting of fibers in a rigid matrix; 3 - twisting of fibers in a pliable matrix.

If the acting stress $\bar{\sigma}$ is comparable with the critical stress of a local protrusion s^*, then the material, especially in the presence of much twisting or at elevated temperatures, when a severe softening of the polymeric matrix is observed, can behave unevenly during extension and contraction, i.e., it becomes "of varying modulus." Naturally, the effect of the amount of twisting is more, the lower s^* is. Hence the danger of a lasting loss of stability, especially for constructions from reinforced plastic whose polymeric matrix is given to a great deal of creep flow. If the lasting shear modulus is sufficiently small, then the gradual development of a local protrusion can be expected under comparatively small contracting stresses. In this connection it is necessary also to take into account the danger shown by V. V. Bolotin and Ye. N. Sinitsyn of a partial protrusion of the reinforcing strands, specifically "surface" protrusion. The quantity which characterizes the number of layers through the total depth of which proceeds substantial protrusion increases with an increase of the contracting forces, the length of the wave of protrusion and the Poisson factor of the binding layers [55].

For materials which weakly resist shear it is essential that during elongation in the direction of reinforcement of a substance whose reinforcing layers have initial irregularities, in the binding appear tangential stresses. They are determined according to the formula [47]

$$\langle \tau_{xz} \rangle^2 = \psi^2_{\sim} \sigma_x^2, \tag{1.3.5}$$

Taking into account the low strength of the polymeric binding, it can be expected that because of the greater twisting of fibers during elongation failure from shear will occur earlier than failure from normal stresses.[1] In this way the twisting of fibers can cause not only a substantial change in the elastic characteristics in the direction of reinforcement, but also a pronounced drop of the strength of a material during loading in the direction of reinforcement.

1.3.3. *Reinforced substance with orderly twisting of the fibers.* For a qualitative evaluation of the effect of small initial distortions especially for materials with a laminate arrangement, another, simpler means of obtaining the relationship for elastic constants allowing for the twisting of reinforcing fibers can be proposed based upon introducing the assumption of an equidistant arrangement of the curved layers. In such an approach the amount of the bend of fibers can be described by a quantity, more simply than by yielding to measurement and an experimental estimate. Imagine that a rod of the material reinforced by the fibers consists of the many equidistant layers twisted (distorted) sinusoidally (Fig. 1.3.2), and it is drawn out evenly by distributed normal forces on two opposite sides. Inasmuch as the twisting of the reinforcing fibers can have not only a random but also an orderly character, with such a model it is possible to describe the behavior of laminate substances with orderly twisting of the reinforcing fibers not only qualitatively, but also quantitatively [167]. Note that by an analogous way a solution can be obtained for any form of equidistant distortion of layers.

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[1] I. G. Zhigun established the values of the critical angle for a series of unidirectional glass-fiber reinforced plastics [102].

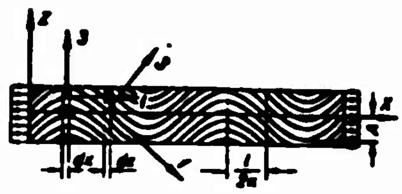

Fig. 1.3.2. A stratified rod with layers twisted (distorted) sinusoidally.

Using this working scheme, each element of length consists of a large number of layers whose orientation depends on the position of the considered element on the axis of the rod, and it can be replaced by a uniform anisotropic substance. The entire rod, consequently, can be considered as an anisotropic substance with directions of the main axes of symmetry 1, 3 continuously changing along the axis. The idea about the possibility of formal averaging of the expression for the modulus of elasticity E_x of a uniform anisotropic material has been expressed in [47].

The modulus of elasticity in the direction of the x-axis for an element of length dx can be determined according to a known formula (see, for example, [123]):

$$\frac{1}{E^{\circ}_x}=\frac{\cos^4\varphi_{1x}}{E_1}+\left(\frac{1}{G_{13}}-\frac{2\nu_{31}}{E_1}\right)\sin^2\varphi_{1x}\cos^2\varphi_{1x}+\frac{\sin^4\varphi_{1x}}{E_3} \qquad (1.3.6)$$

(ϕ_{1x} - angle formed by direction 1 with the x-axis).

For sinusoidally twisted fibers

$$\varphi_{1x}=\operatorname{arctg}\left[A\frac{k\pi}{l}\cos\frac{k\pi}{l}x\right].$$

We will consider that section perpendicular to the axes remain flat.[1] Then averaging by means of integrating expression (1.3.6) the deformation along the axis of the rod over a length which is

[1]An accurate solution to the problem could be obtained by substituting into the general equations for the plane problem of the theory of elasticity of anisotropic media elastic constants in the form of functions of ϕ_{1x}. However, this would lead to unjustified mathematical difficulties. Therefore for the considered problem, whose basis is a very approximate model, an approximate solution based upon the hypothesis of plane sections is preferable.

figured in multiples of a quarter-period of a sine curve or much greater, we obtain in closed form the expression for the apparent modulus along the x-axis:

$$\frac{1}{E_x}=\frac{1}{E_1}\cdot\frac{2+f_{\sim}^2}{2(1+f_{\sim}^2)^{3/2}}+\left(\frac{1}{G_{13}}-\frac{\nu_{31}}{E_1}\right)\frac{f_{\sim}^2}{2(1+f_{\sim}^2)^{3/2}}+$$
$$+\frac{1}{E_3}\left[1-\frac{2+3f_{\sim}^2}{2(1+f_{\sim}^2)^{3/2}}\right] \qquad (1.3.7)$$

($f_{\sim}=A\frac{\pi k}{l}$ - parameter characterizing the distortion of fibers; k - number of half-waves on the base l, A - amplitude of sine curve).

Relationship (1.3.7) is illustrated by a graph (Fig. 1.3.3) showing the ratio $\frac{E_x}{E_1}$ as a function of the amount of curvature of the layers $f_{\sim}$. From the graph it is evident that for large $f_{\sim}$, when the plies have been arched into a "bellows," as one would expect, $E_x \to E_3$. The initial section of relationship $E_x(f_{\sim})$ with small curvature of the strand (small $f_{\sim}$) with sufficient accuracy is described by the expression

$$E_x=\frac{E_1}{1+\frac{E_1}{G_{13}}\cdot\frac{f_{\sim}^2}{2}}. \qquad (1.3.8)$$

This section is shown on Fig. 1.3.4 (curves 1, 2, 3, 4, 5 have been constructed for the same relationships of elastic constants). Even at comparatively small twisting the modulus of elasticity E_x in real materials can be considerably lower than the modulus of elasticity E_1 for a material with ideally straight fibers. As it appears from expression (1.3.8), this is a consequence of weak resistance of the material to shear.

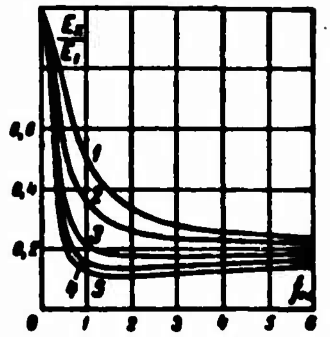

Fig. 1.3.3. Dependence of the modulus of elasticity in the direction of strands upon the amount of their distortion. Curves 1-5 have been constructed for $\frac{E_1}{G_{13}} = 5,\ 10,\ 20,\ 30,\ 40;\ \frac{E_3}{E_1} = 0.2$.

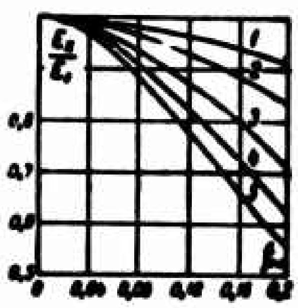

Fig. 1.3.4. Initial section of the dependence of the modulus of elasticity upon the distortion of filaments. Designations are the same as in Fig. 1.3.3.

Relationship (1.3.8), describing the effect of orderly distortions of strands on the modulus of elasticity in the direction of reinforcement, accurate to within the designations agrees with expression (1.3.2), derived from a substance with random distortions. This testifies to the fact that a material with sinusoidal twisting of filaments during deformation along the fibers is equivalent to a material with random twisting of the fibers, for which $\psi_{\sim} = \frac{l_{\sim}}{\sqrt{2}}$.

For determination of the modulus of elasticity in the direction of the z-axis we examine contraction (elongation) in this direction of a rod with sinusoidal twisting of fibers along the x-axis, and forces, evenly distributed on its ends. The dimensions of the rod in the direction of the z-axis are assumed to be rather large, but in the direction of the x-axis is confined to no less than two half-waves of a sine curve. The expression for the modulus of elasticity in the direction of the z-axis for an element of length dx is determined according to the formula (see, for example, [123])

$$\frac{1}{E^*_z} = \frac{\sin^4 \varphi_{1z}}{E_1} + \left(\frac{1}{G_{13}} - \frac{\nu_{31}}{E_3}\right) \sin^2 \varphi_{1z} \cos^2 \varphi_{1z} + \frac{\cos^4 \varphi_{1z}}{E_3}. \qquad (1.3.9)$$

If we disregard the distortion of sections perpendicular to the z-axis and average along the z-axis the deformation of all elements dx in the direction of the x-axis, then we obtain the following expression for the apparent modulus of elasticity in the direction of the z-axis:

$$\frac{1}{E_z} = \frac{1}{E_1}\left[1 - \frac{2+3f^2_\sim}{2(1+f^2_\sim)^{3/2}}\right] + \left(\frac{1}{G_{13}} - \frac{2\nu_{31}}{E_1}\right)\frac{f^2_\sim}{2(1+f^2_\sim)^{3/2}} +$$

$$+ \frac{1}{E_3} \cdot \frac{2+f^2_\sim}{2(1+f^2_\sim)^{3/2}}. \qquad (1.3.10)$$

At $f^2_\sim \ll 1$ relationship (1.3.10) can be rewritten as follows

$$E_z = \frac{E_3}{1 + \frac{E_3}{G_{13}} \cdot \frac{f^2_\sim}{2}}. \qquad (1.3.11)$$

from which it is clear that deformation in the direction perpendicular to reinforcement is affected insignificantly by the curvature of the plies, since G_{13} and E_3 are quantities of one order. It is possible to show analogously that the shear modulus G_{xz} of a substance with small initial irregularities equal to

$$G_{xz} = \frac{G_{13}}{1 + 2\left(\frac{G_{13}}{E_3} - 1\right) f^2_\sim}. \qquad (1.3.12)$$

depends little on the twisting of reinforcing layers.

The twisting of the reinforcement affects the degree of anistropy of the elastic properties. This can be shown in the example of the

ratio $\frac{E_1}{G_{zz}}=\beta^2$, if we use expressions (1.3.8) and (1.3.12) and take into account that for small twisting $G_{xz}\approx G_{13}$. In this instance the parameter of anisotropy β for a material with twisted fibers can be determined by the formula

$$\beta^2=\bar{\beta}^2-\frac{f^2}{2}\bar{\beta}^4; \tag{1.3.13}$$

$\bar{\beta}=\sqrt{\frac{E_1}{G_{13}}}$ characterizes the degree of anisotropy of a material with ideally straight fibers. As it appears from the presented data, small distortions of the fibers are the reason for the considerable change in elastic and strength properties in the direction of reinforcement, but at the same time they have practically no effect on properties determined by a pliable matrix.[1] Materials with straightened fibers possess a greater sensitivity to shear as a result of a growth of the anisotropy parameter. These conclusions subsequently will be confirmed on glass-fiber reinforced plastics reinforced by roving and fabric.

1.3.4. *An experimental check.* An estimate of the accuracy of formulas (1.3.8) and (1.3.12) has been made in samples from a unidirectional material with the earlier assigned sinusoidal distortion of the reinforcing layers (Fig. 1.3.5b). The samples were prepared on the device described earlier (Fig. 1.2.4), on the basis of the mold of which with the help of simple implements it was possible to obtain samples with a different degree of twisting of fibers. The principle of preparation of sample strips is clear from Fig. 1.3.5a. Differences in the degree of curved strands with respect to amplitude and frequency were created with the help of special inserts fastened to the punch and die of the mold. In ready samples the projections formed by the inserts are removed. In the preparation of samples with preassigned twisting of the fibers the height of the projections on the inserts (Fig. 1.3.5a) was 6 mm, and

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[1]The expressions which describe the effect of orderly initial distortions on the Poisson factors have been omitted. They are given in a work of I. G. Zhigun [101]. The same book gives equations which describe the effect of orderly distortions $\nu_{xy}, \nu_{yz}, \nu_{xz}$ of laminate and three-dimensional cross-linked materials, and makes numerical evaluations.

the distance between the projections 10, 20, 30 and 40 mm. When the projections were every 10 mm after pressing the samples had practically straight fibers. For remaining distances between the projections the amplitudes of the distortions were respectively 1.0, 1.2 and 1.5 mm; the length of the wave remained unchanged after the preparation of the samples. Laminations and cracks in the depth were not observed.

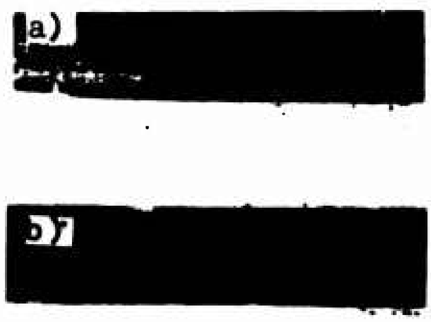


Fig. 1.3.5. Samples with orderly curved fibers: a) arrangement of reinforcing layers during the preparation of samples (implement has projections forming the preassigned distortion of fibers); b) experimental samples with fibers curved sinusoidally (macrograph, mag. 2×).

For contrast, besides samples with preassigned orderly curvature of fibers created intentional, samples with fibers preliminarily straightened but unstressed during pressing were made and tested. Before pressing the glass fibers were stretched in order to guarantee their straightening and orientation along the axis of the samples. Then the tension was removed. The twisting of fibers of this type of samples was the only consequence of thermal and chemical contraction of the binding. The absence in the mold of walls perpendicular to the direction of the preliminarily oriented glass fibers prevented their twisting during contraction by the punch. The possibility of distortion of the fibers as a result of contraction of binding was retained.

During the preparation of samples of both types the technological regime (temperature, time and pressure) was rigidly maintained. Samples were made from one batch of AG-4S material. In order to exclude the effect of relaxation processes the modulus of elasticity

was determined thirty days after the preparation of the samples. Samples with preassigned twisting of the fibers were in the shape of a rectangular prism (280 × 10 × 10 mm^3), and with straightened but unstressed fibers - strips (280 × 20 × 3 mm^3).

Both types of samples were used for the estimation of the effect of preassigned twisting of fibers on the modulus of elasticity in the direction of the fibers and the modulus of interlamination shear (i.e., one characteristic determined basically by fibers $E_x{}^+$, was selected, and one transverse characteristic determined mainly by the polymeric matrix G_{xz}). Experimental results are presented in Tables 1.3.1 and 1.3.2. These tables include also the results of a calculation using formulas (1.3.8) and (1.3.12). Values of the remaining elastic constants used in the calculation are given in the footnotes to the tables.

Table 1.3.1. The effect of preassigned twisting of fibers on the modulus of elasticity.

Modulus of elasticity ($\cdot 10^5$), kgf/cm^2		Straightened unstressed fibers $E_x{}^0$	Degree of the twisting of fibers, $f_\sim$		
			0.078	0.118	0.157
E_x, from (1.3.8)*		—	3.31—3.57	3.02—3.12	2.68—2.83
Experiment	$E_x{}^+$	3.6	3.33	3.00	2.75
	$\frac{E_{x\,min}}{E_{x\,max}}$	—	$\frac{3.08}{3.54}$	$\frac{2.80}{3.45}$	$\frac{2.11}{2.81}$
	v_E	—	5.15	6.3	7.1
	n	—	10	10	10
$\frac{E_x{}^f}{E_x{}^0}$		1.00	0.925	0.833	0.765

*Numerical values of E_x from formula (1.3.8) have been calculated for E_1 = 3.6·10^5 kgf/cm^2 and $\frac{E_1}{G_{13}}$ = 22-28, determined for AG-4S material from independent experiments. $E_x{}^f$ - values of the modulus of elasticity of samples with preassigned twisting of fibers.

Table 1.3.2. The effect of preassigned twisting of fibers on the shear modulus.

Modulus of elasticity ($\cdot 10^{6}$) kgf/cm^2		Straightened unstressed fibers G_{xz}^{0}	Degree of the twisting of fibers, $f_{\sim}$	
			0,078	0,157
G_{xz}, from (1.3.12)*		0,143	0,147	0,151
Experiment	G_{xz}	0,143	0,149	0,141
	$\frac{G_{xz\,min}}{G_{xz\,max}}$	$\frac{0,112}{0,179}$	$\frac{0,129}{0,185}$	$\frac{0,116}{0,182}$
	v	14,4	16	17
	n	10	8	8
$\frac{G_{xz}^{f}}{G_{xz}^{0}}$		1,00	1,04	0,98

* G_{xz}^{f} – the value of the modulus of interlamination shear for a material with preassigned twisting of fibers. In calculation it is assumed that $E_3 = 0.672 \cdot 10^5$ kgf/cm^2.

Table 1.3.1 contains the results of an experiment on the determination of the modulus of elasticity in the direction of the fibers. Tests were conducted during elongation; specific attention was given to the choice of the dimensions of the sample, since the unsuccessful choice of a gauging base during tests on elongation can lead to the inaccurate estimation of the modulus of elasticity. For visualization the change in modulus depending on the degree of distortion of the reinforcement is presented in Fig. 1.3.6 (curve 1); the zero point in the figure corresponds to the preliminarily stressed samples (Table 1.4.1), during the preparation of which the danger of twisting the strands is removed.

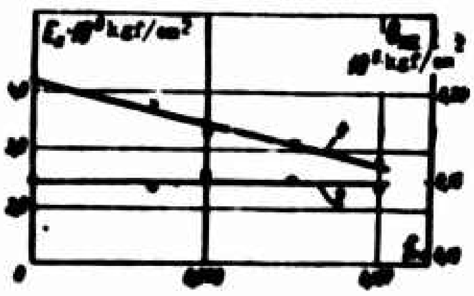


Fig. 1.3.6. Effect of the degree of distortion of fibers on E_x^{+} and G_{xz} (unidirectional AG-4S material): 1 - E_x^{+}; 2 - G_{xz} .

Comparison of theoretical and experimental data shows good agreement of the simplified theory with experiment. The data also indicate the earlier noted substantial effect of the twisting of fibers on the modulus of elasticity in the direction of reinforcement.

Samples with preassigned twisting of fibers (earlier tested for elongation) were also used for estimating the effect of the twisting of fibers on the modulus of interlamination shear G_{xz}. A direct determination of G_{xz} is extremely difficult, therefore shear modulus was established indirectly during tests on the curvature of beams by a force applied halfway along the span with a different ratio $\frac{H}{l}$ (height:span). The effect of shears on bending deflection w_{max} was estimated according to a formula of S. P. Timoshenko [239] (see § 2.3).

$$w_{max} = w^{\circ}_{max}\left[1+\frac{6}{5}\cdot\frac{H^2}{l^2}\cdot\frac{E_x}{G_{xz}}\right], \qquad (1.3.14)$$

where $w^{\circ}_{max}=\frac{Pl^3}{48E_xJ}$ - bending deflection not allowing for shear; J - moment of inertia. A procedure of determining G_{xz} from these means is presented, for example, in [186, 256]. Obtained data are put into Table 1.3.2 and represented in Fig. 1.3.6 (curve 2). The zero point in Fig. 1.3.6 is obtained in samples with preliminarily stressed fibers, where the danger of distortion has been removed. This table includes results obtained during tests of samples with preliminarily straightened unstressed fibers. Dispersion analysis (with 95% reliability) verifies that small distortions of strands has practically no influence on the modulus of interlamination shear G_{xz}. Comparison of experimental data with that calculated by formula (1.3.12) shows good agreement of results: values of E_3 and G_{13} necessary for calculation are found from formulas [45].

Earlier it was noted that as a result of low strength of the polymeric binding one should expect that at sufficiently great distortion of fibers during elongation failure from shear will occur

earlier than failure from normal stresses. For an experimental check of this assumption 18 samples were tested with the above described three levels of distortion of sinusoidally curved layers and samples with the random twisting of fibers. All samples were tested in shear. The nature of the failure is observable in Fig. 1.3.7. Normal stresses corresponding to failure lie over the range from 11 to 30 kgf/mm^2. Note that the tensile strength of samples with completely straightened fibers from the same batch of material constitutes 72 kgf/mm^2.

NOT REPRODUCIBLE

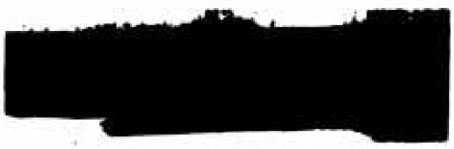


Fig. 1.3.7. Failure during elongation of samples with much local distortion of fibers.

In this way, the fact coming from theoretical considerations that small distortions of the fibers, which can appear in articles are the source of a substantial change in the modulus of elasticity in the direction of reinforcement and strength during elongation in the direction of the fibers and little affect shear rigidity is experimentally confirmed.

1.3.5. *Extension of an orderly model to woven materials.* The equations given earlier considering the effect of distortion of fibers are obtained for unidirectional materials. Bazhant [24], using the method of the "reduced section" [227], extended the orderly model to woven materials of linen interweaving. It turned out that the effect of twisting of fibers on the modulus of elasticity in the direction of the warp (or woof) is described by the following approximation expression:

$$E_x = E'(1-\chi_x) + \frac{E'\chi_x}{1+\dfrac{\frac{\pi}{4}}{1+\frac{G_{12}}{G_{13}}\cdot\frac{l_1^2}{l_2^2}}\chi_x\frac{A^2}{l^2}\cdot\frac{E'}{G_{13}}}. \qquad (1.3.15)$$

where E' and E'' - moduli of elasticity of reinforcement and of polymeric matrix; χ_x and $(1-\chi_x)$ - volume content of reinforcement and polymeric binding in direction x; l_x and l_y - length of waves respectively in direction of x and y; A and l - maximum amplitude and half-wave length.

For equilibrium materials (1:1 arrangement) with a high percent content of reinforcement relationship (1.3.15) assumes the form

$$E_x \approx E''\frac{1-\chi}{2}+\frac{E'\frac{\chi}{2}}{1+\frac{\pi^2}{2}\cdot\frac{A^2}{l^2}\cdot\frac{\chi}{2}\cdot\frac{E'}{G_{13}}} \quad (1.3.16)$$

considering, that $G_{23}=G_{13}$, $l_x=l_y$ and $\chi_x=\chi_y=\frac{\chi}{2}$.

Expression (1.3.16) allows evaluating the effect of small initial distortions in the case when the properties of components are known and also their percent content in a material. If we take $E'\frac{\chi}{2}\approx E_1$ and disregard the contribution from the polymeric binding, we obtain

$$E_x \approx \frac{E_1}{1+\frac{\pi^2}{2}\cdot\frac{A^2}{l^2}\cdot\frac{E_1}{G_{13}}} \approx \frac{E_1}{1+\frac{E_1}{G_{13}}\cdot\frac{f^2}{2}},$$

i.e., the expression which describes the effect of initial distortions in the case when the parameters of an ideal model are known.

Experiments on glass-fiber reinforced plastic of balanced structure SKT-11 (Table 1.3.3) showed good agreement with formulas (1.3.8) and (1.3.16) [101]. The degree of distortion (wave amplitude and length) was changed by stretching the strands in the process of preparing the samples. These parameters were measured on a general-utility microscope. The characteristics of an ideal model and the values of the constant components of a material and their percent content in the material are given in the footnotes to Table 1.3.3.

Table 1.3.3. The effect of initial twisting of fibers on the elastic characteristics of woven materials.

Working* relationships and experimental data	Stretching along warp					Testing along warp and woof**
	testing on warp		testing on woof			
	Degree of preliminary stretching, fractions of H_{gr}					
	0	0,1	0	0,25	0,5	0,1
Data of experiment	1,28	1,68	1,35	1,14	1,09	1,61
Calculated by (1.3.8)	1,30	1,77	1,30	1,00	1,02	1,53
Calculated by (1.3.16)	1,42	1,77	1,42	1,12	1,07	1,56
A, mm	0,135	0,055	0,135	0,190	0,200	0,11
l, mm	2,5	2,55	2,50	2,43	2,40	2,22

*During calculation according to (1.3.8) the quantities $E_x = 1.87 \cdot 10^5$ kgf/cm² are determined from independent experiments in samples with straightened reinforcements. In calculation by (1.3.16) $E' = 5.1 \cdot 10^{5}$ kgf/cm², $E'' = 0.21 \cdot 10^{5}$ kgf/cm².

**Samples tested on the warp.

§ 1.4. Preliminary Tension of Reinforcement

1.4.1. *Production-process stretching of fibers.* From the theory given in the previous section, it follows that to improve in the products the characteristics of the examined materials it is necessary to remove or, at least, to decrease the danger of the distortion of fibers in the formation process. For this purpose during the preparation of constructions from materials reinforced by fibers production-process stretching of the reinforcement is used. The analytic relationships which describe departure from the ideal model contain the parameter $f_{\sim}$. The magnitude of this parameter, and consequently, the values of the elastic and strength properties of all considered types of structures in one or another measure change depending on the tension.

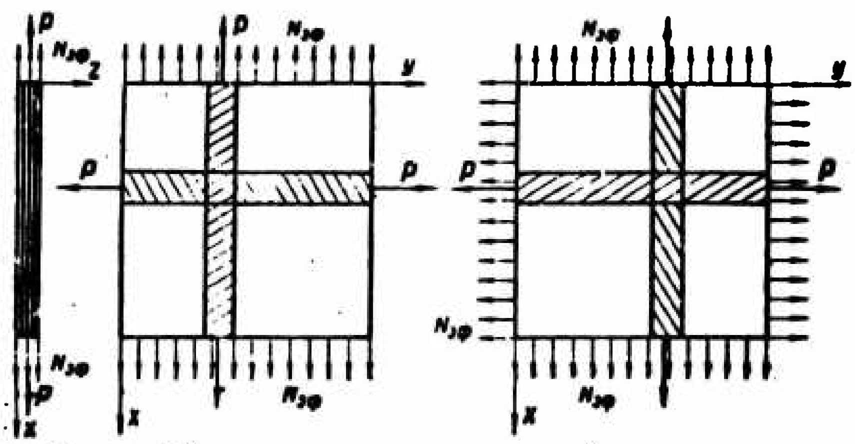
The technology of the preparation of products with the stretching of fibers has a number of features conditioned by the existing designs of the stretching devices. In making parts from nonwoven

materials the tension matches the direction of reinforcement. The technology of the preparation of preliminarily stressed constructions (plane and three-dimensional) from woven materials in such that the tension is applied to the fibers of the warp or to the fibers of woof. Naturally, tension in one direction is connected with a change in the arrangement of reinforcing fabric in the other direction. Tension of the reinforcement in two mutually perpendicular directions is technologically very difficult to attain. The necessity of changing to this means of preparation depends on the obtained increase in rigidity and the strength of the constructions made with tension on the warp and woof. However, efforts in this area are practically nonexistant. Basically the effect of tension on the property of unidirectional materials is being studied, for fabrics mainly the effect of tension on the warp on properties in this direction has been examined.

Experimental data allow, in the example of glass-fiber reinforced plastics, evaluating at least qualitatively the change in strength and elastic characteristics of composite materials of different structure depending on the tension, and establishing the optimum amount of this force. I. G. Zhigun studied [102] the effect of tension on strength and rigidity during elongation, contraction,[1] shear and bending of oriented glass-fiber reinforced plastics, reinforced by roving and fabric with usual and three-dimensional interweaving of the fibers. Although experiments have been made on one (at most on two) brand of each type of material (unidirectional AG-4S and LSB-F materials, glass fiber laminates SKT-11 on a phenol-formaldehyde binding, VPS-7 on epoxy binding 3DT-10 and one type has been of three-dimensional cross-linked structures on phenol-formaldehyde and epoxy binding), obtained data can be considered as typical for reinforced materials.

[1]A model proposed by Rozen [190] is completely inexact, since it does not take into account initial distortion of fibers.

Experiments have been carried out on plane and circular samples (coordinate axes, variants of the tension of filaments, pattern of cutting the samples and the direction in which tests were made are indicated in Fig. 1.4.1). Plane specimens were made in the form of strips (Fig. 1.4.1a) or plates (Fig. 1.4.1b, c), which then were cut into samples of preassigned dimensions. Coordinate axes coincide with the direction of the reinforcing strands. In unidirectional materials the x-axis coincides with the direction of reinforcement; in woven materials the x-axis matches the direction of the warp, and the y-axes with the direction of the woof. For convenience of describing the examined combinations of tension and the direction of the tests of the samples the following designations are introduced: $E^{+(-)}_{x(0)}$, $E^{+(-)}_{x(x,y)}$, $G_{xz(x)}$ and $\Pi^{+(-)}_{y(x,0)}$, etc. (the upper index indicates elongation (+) or contraction (-); the lower indicates the directions of the tests, which match the direction along which the sample has been cut; the parentheses are the indexes which characterize the presence and direction of preliminary tension of the strands). The strip samples have one index (for example, $E^{+}_{x(x)}$), and samples cut from plates have two (for example, $E^{+}_{y(0,y)}$ means the modulus of elasticity during elongation in the direction of the woof, measured in a sample cut from a plate made under conditions when the strands of woof were preliminarily stressed and there was no tension in the direction of the warp).



a) b) c)

Fig. 1.4.1. Production-process stretching of strands and of the cutting of forms. a) samples from unidirectional materials; b) plate from woven materials (tension on warp); c) plate from woven materials (tension on warp and woof).

Subsequently data are given about the effect of tension on deformation characteristics necessary for the solution to the problem making up the basic content of the book (rods, plates, thick-walled rings from unidirectional and woven materials). On strip samples the effect of tension on $E_x^{+(-)}$ and G_{xz} was examined, and combinations $E_{x(0)}^{+(-)}, E_{x(x)}^{+(-)}$, $G_{xz(0)}$ and $G_{xz(x)}$ were studied. On samples cut out from plates $E_x^{+(-)}$, $E_y^{+(-)}$, G_{xz}, $\nu_{yx}^{+(-)}$ were examined (the examined combinations are given in Table 1.4.1). These samples were used for research on the effect straightening and stretching strands on strength during elongation, contraction and bending.

Table 1.4.1. Examined deformation characteristics of plates.*

Preliminary tension		Straightened unstressed fibers	Tension on warp and woof	Tension on warp	Tension on woof
Deformation characteristic	E_x	$E_x^{+(-)}{}_{(0,0)}$	$E_x^{+(-)}{}_{(x,y)}$	$E_x^{+(-)}{}_{(x,0)}$	$E_x^{+(-)}{}_{(0,y)}$
	E_y	$E_y^{+(-)}{}_{(0,0)}$	$E_y^{+(-)}{}_{(x,y)}$	$E_y^{+(-)}{}_{(x,0)}$	$E_y^{+(-)}{}_{(0,y)}$
	G_{xz}*	$G_{xz(0,0)}$	$G_{xz(x,y)}$	$G_{xz(x,0)}$	$G_{xz(0,y)}$
	ν_{yx}	$\nu_{yx}^{+(-)}{}_{(0,0)}$	$\nu_{yx}^{\pm(-)}{}_{(x,y)}$	$\nu_{yx}^{+(-)}{}_{(x,0)}$	$\nu_{yx}^{+(-)}{}_{(0,y)}$

*For the examined materials the effect of tension on G_{yz} is near G_{xz}.

1.4.2. *Samples and means of testing them.* The choice of form and dimensions of a sample for investigating the effect of tension on strength and rigidity of materials reinforced by fibers was made allowing for the characteristics of these materials. Samples and measuring base were selected allowing for the zone of boundary effect. The modulus of elasticity and strength during elongation in the direction of strands are examined on strip samples.[1] As was indicated in [227], such a form of the samples is rational during elongation tests of glass-fiber reinforced plastics. The base for measurement of the modulus of elasticity was located at a distance not less than six thicknesses of the sample from the place of the loading. Samples were held in the grips by friction.

[1]Features of shear tests of glass-fiber reinforced plastics have been studied by Ya. S. Sidorin [195, 196].

For measuring deformation resistance strain gauges and the deformation converter described in [263] were used. Deformation was measured on two sides of a sample with the help of a deformation gauge or were recorded on oscillograph film. The measurement of deformation was in an area of stress exceeding $0.5\Pi_{в}^{+}$ ($\Pi_{в}^{+}$ - strength during elongation), i.e., in the linear section of dependence $\sigma \sim \varepsilon$ up to the beginning of crack formation in the material; deformation curves are given later in Fig. 1.4.7.

Contraction tests in standard prismatic samples naturally give incorrect results. The basis when selecting the size and dimensions of a sample and the device for contraction tests was an American standard ASTM-D 695T [271]. From this standard a sample in the form of a lamina (depth 2-4 mm) was placed between the clamps of the device to keep it steady (Fig. 1.4.2). Deformation was determined with resistance strain gauges fixed to the middle part of the sample. The deformed state in this part of the sample was nearly uniaxial. The $\sigma - \varepsilon$ relationship during contraction is shown also in Fig. 1.4.7.

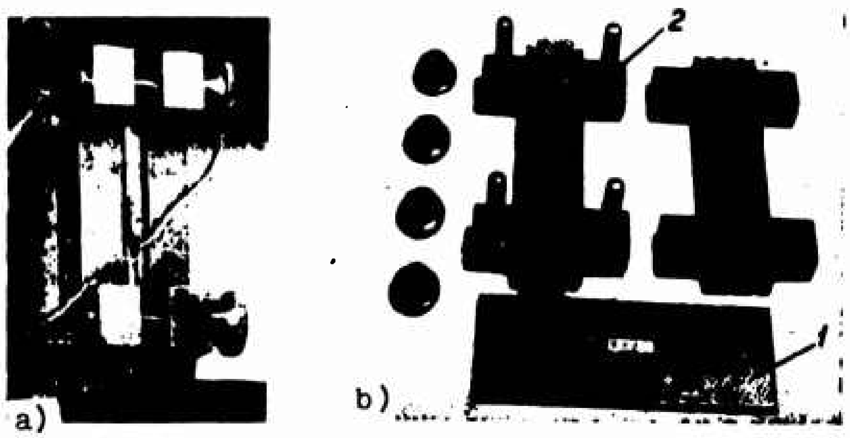


Fig. 1.4.2. Device for contraction tests.
a) in the collection; b) parts of device and experimental samples for determination of elastic (1) and strength (2) characteristics.

The mechanical features of materials reinforced by fibers and intended for processing into products by the winding method are examined, as a rule, in circular samples (see, for example, [284, 313], loaded using two half rings (so-called NOL-ring method). In this case it is considered that a uniform pressure on the entire interior surface of the ring has been created. Review [266] generalizes an experiment of Plastics Research Inst. TNO (Delft, Holland). By contrasting various methods of winding materials (on half-rings, tests on bending by concentrated forces, tests of sections on interlamination shear and bending, test of rings with cuts, etc.) it has been shown that on thin rings the NOL-ring method gives good results for the estimation of strength during elongation (characteristic data are given in Fig. 1.4.3) and resistance to interlamination shear.

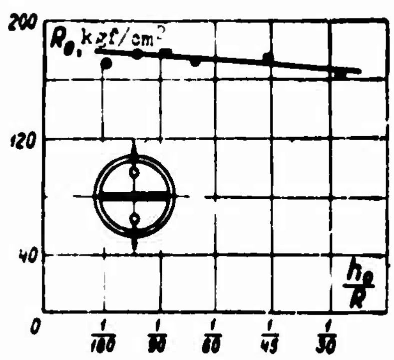


Fig. 1.4.3. Dependence of apparent strength during elongation upon ring depth [266] (tests on half-disks; $\frac{h_0}{R}$ - relative thickness; R_θ - strength in the direction of fibers).

A change to large relative thicknesses ([266] examines the range $\frac{h_0}{R}=\frac{1}{180}-\frac{1}{30}$) and the strengthening of a material in the direction of reinforcement led to the necessity of searching for other methods of loading (one of them is indicated in Fig. 1.4.4). Circular samples were tested in a special device for elongation under the action of interval pressure. The principle of action of the device is clear from the figure. Pressure was induced on the sample by a rubber ring. The calibrating was done by measuring the deformation of the steel force-measuring ring with strain gauges in place

of the experimental ring. It has been established that to the experimental ring is transferred 0.95 of the preassigned pressure. The modulus of elasticity is measured with the help of a special device (Fig. 1.4.5), also recording the deformation of the external surface.

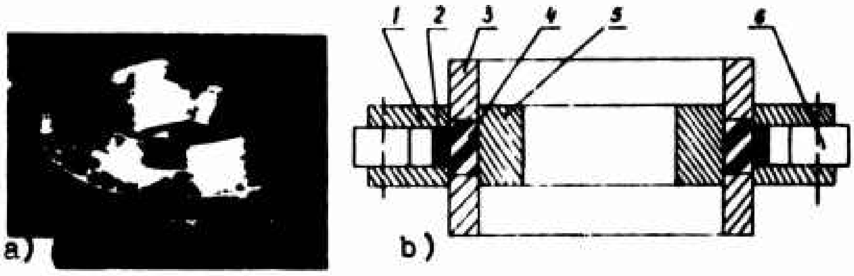


Fig. 1.4.4. Device for testing interior pressure. a) overall view; b) loading scheme: 1 - external plates; 2 - sample; 3 - pressure rings; 4 - rubber ring; 5 - interior ring; 6 - spacer layers.

Fig. 1.4.5. Device for the measurement of radial deformation: 1 - experimental ring; 2 - movement pickup.

1.4.3. *The effect of tension on rigidity.* The experimental curves which characterize the change in modulus of elasticity E_x depending on the tension of fibers reinforced by plastics with unidirectional, laminate and three-dimensional cross-linked structures are presented in Fig. 1.4.6. They were obtained during tests on elongation in the direction of reinforcement of all three types of samples. The effective force of tension (allowing for a drop during formation) is indicated in fractions of strength of material $П_м$ in the processing state. The estimation was made at a level of stress not exceeding 0.5 of the strength during extension-contraction. Typical curves are shown in Fig. 1.4.7.

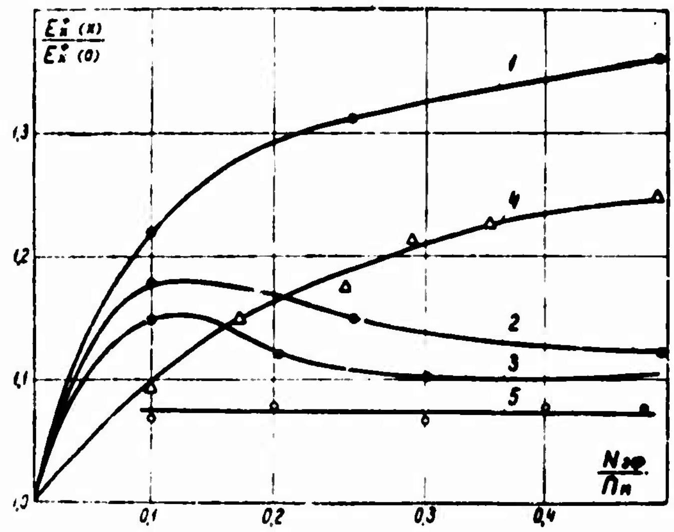


Fig. 1.4.6. The effect of tension on the modulus of elasticity in the direction of fibers: 1 - SKT-11 (laminate, strips, tests along warp), 2 - AG-4S (unidirectional, strips), 3 - 3DT-10 (volume weave, strips and plates, tests in the direction of the warp), 4 - VPS-7 (laminate, ring, tests in the direction of the warp), 5 - LSB-F (unidirectional, ring).

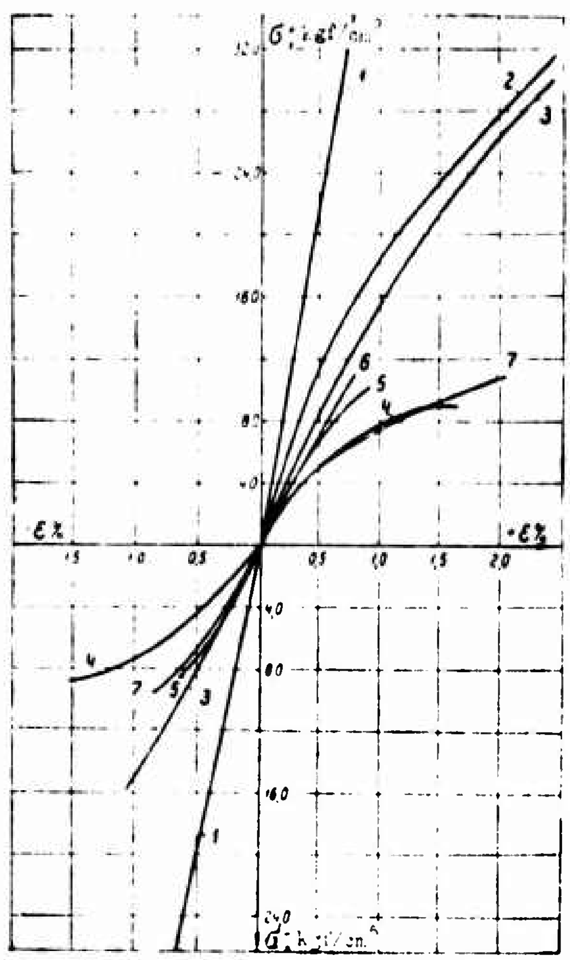


Fig. 1.4.7. Relationship between stress and deformations for glass-fiber reinforced plastics of various structure: 1 - unidirectional material AG-4S; 2, 3, 4 - laminate material SKT-11 (loading on warp: 2 - tension on warp equal to 0.2 Π_M, 3 - without tension, 4 - loading on woof, at a tension on the warp of 0.5 Π_M); 5, 6, 7 - three-dimensional cross-linked material (loading on warp: 5 - angle of inclination of reinforcement $\omega_0=36°$, 6 - $\omega_0=19°$; 7 - loading on woof, $\omega_0=19°$).

The behavior of unidirectional materials during tension has been examined in the example of a typical representative of this kind of materials - glass-fiber reinforced plastic AG-4S. The data (Table 1.4.2) indicate that the tension of fibers leads to an increase in the modulus of elasticity $E_x^{+(-)}$ not only in comparison with the amount obtained on full-scale articles, but also in comparison with samples with straightened but unstressed fibers made in the same pressmold. Dispersion analysis indicated that with a reliability of not less than 0.95 this difference is significant. The effect of tension on elastic constants is exhibited only in the detection of reinforcing fibers. The amount of tension must be sufficient in order to avoid distortions due to production process contraction of the material in the formation process, i.e., $N_{н\phi} > N_{кр}$ ($N_{кр}$ - force of tension necessary to prevent distortion of fibers and depending on the material used and the conditions of formation). As experiments (Table 1.4.2) showed, to get the maximum rigidity of glass-fiber reinforced plastic AG-4S a comparatively low tension is sufficient - not more than 0.1-0.15 Π_M. The further increase in the force of a tension no longer affects the modulus of elasticity $E_x^{+(-)}$ for unidirectional materials. The difference in moduli at $0{,}1\Pi_M$, $0{,}2\Pi_M$, $0{,}3\Pi_M$ and $0{,}4\Pi_M$ is random (at the same level of confidence).

Tension affects differently the properties of woven materials. In reinforced plastics with a laminate structure in the examined range of change $N_{н\phi}$ an increase of tension continuously increases the modulus of elasticity in this direction $\bar{E}_x^{+(-)}{}_{(x,\theta)}$. As it appears from Fig. 1.4.8, this is explained by the continuous decrease (depending on the tension force) of the distortions of fibers of the warp preassigned by the structure of the weave of the reinforcing fabric. Experimental data obtained on SKT-11 material reinforced by fabric of linen interweaving are collected in Table 1.4.3. It has been established that the structure of interweaving of fabric in the plane does not prove to have a fundamental effect on the relationship $\bar{E}_{x(x)} = f(\bar{N}_{н\phi})$ for laminate materials. This is indicated by curves 1 and 4 in Fig. 1.4.6, obtained during tests of plastic reinforced by fabric of linen and sateen interweaving.

Table 1.4.2. The influence of straightening and tension of fibers on the modulus of elasticity $E_x^{+(-)}$ of unidirectional materials (AG-4S).

Modulus of elasticity ($\cdot 10^5$), kgf/cm^2	Technological twisting of fibers	Straightened unstressed fibers	Preliminary tension of fibers in fractions of Π_M			
			0.1	0.2	0.3	0.4
E_x^+	2,8–3,30	3,6	4,1	3,9	4,0	3,8
$\frac{E_{min}^+}{E_{max}^+}$	—	$\frac{3,2}{4,0}$	$\frac{3,8}{4,5}$	$\frac{3,6}{4,5}$	$\frac{3,5}{4,5}$	$\frac{3,4}{4,2}$
v_E (%)	—	5,81	6,37	8,55	13,6	7,35
n	7	12	10	10	7	6
$\frac{E_N}{E_0}$	0,78—0,91	1,0	1,14	1,08	1,11	1,06
E_x^-	—	3,5	4,0	3,8	3,9	3,9
$\frac{E_{min}^-}{E_{max}^-}$	—	$\frac{3,2}{3,8}$	$\frac{3,6}{4,5}$	$\frac{3,5}{4,3}$	$\frac{3,5}{4,0}$	$\frac{3,6}{4,2}$
v_E^-, %	—	7,61	11	6	6,27	5,98
n	—	7	6	5	6	5
$\frac{E_N}{E_0}$	—	1,0	1,14	1,12	1,11	1,11
$\frac{E^+}{E^-}$	—	1,025	1,020	1,020	1,020	0,975
G_{xz}	—	0,143	0,146	0,143	0,144	0,139
$\frac{G_{xz\,min}}{G_{xz\,max}}$	—	$\frac{0,112}{0,179}$	$\frac{0,124}{0,175}$	$\frac{0,118}{0,162}$	$\frac{0,115}{0,159}$	$\frac{0,112}{1,160}$
v_E, %	—	14,4	9,6	13,0	12,8	14,6
n	—	10	10	8	8	7
$\frac{G_{xz}^N}{G_{xz}^0}$	—	1,00	1,02	1,00	1,00	0,97

Table 1.4.3. Change in moduli of elasticity and Poisson coefficients of extension-contraction depending on the amount of preliminary tension of the reinforcements.*

Moduli of elasticity and Poisson coefficients ($\cdot 10^5$), kgf/cm²	Tension on warp								Tension on warp and woof	
	Tests on warp				Tests on woof				Tests on warp	
	Preliminary tension of reinforcement, in fractions of Π_M									
	0	0.1	0.25	0.5	0	0.1	0.25	0.5	0.1	0.25
E_x^+	1,38	1,68	1,80	1,87	1,35	1,21	1,14	1 [illegible]0	1,61	1,70
$\frac{E_H}{E_0}$	1,00	1,22	1,31	1,36	1,00	0,89	0,85	0 [illegible]4	1,17	1,23
E_x^-	1,38	1,61	1,72	1,75	1,36	—	—	1, [illegible]5	1,58	1,65
$\frac{E_H^-}{E_0}$	1,00	1,16	1,25	1,29	1,00	—	—	0, [illegible]7	1,16	1,21
$\frac{E^+}{E^-}$	1,0	1,04	1,04	1,07	0,99	—	—	0, [illegible]5	1,02	1,03
ν	0.130	0,143	0,148	0,152	0,133	0,100	0,093	0 [illegible]80	0,136	0,134
$\frac{\nu_H}{\nu_0}$	1,00	1,11	1,17	1,14	1,00	0,78	0,67	[illegible]80	1,04	1,03
$\frac{E_x^+{}_{(y,0)}\nu_{y(x,0)}}{E_y^+{}_{(x,0)}\nu_{x(x,0)}}$	1,04	0,97	0,99	0,96	No data					

* Material SKT-11, preliminary heat treatment of fabric at 250°C.

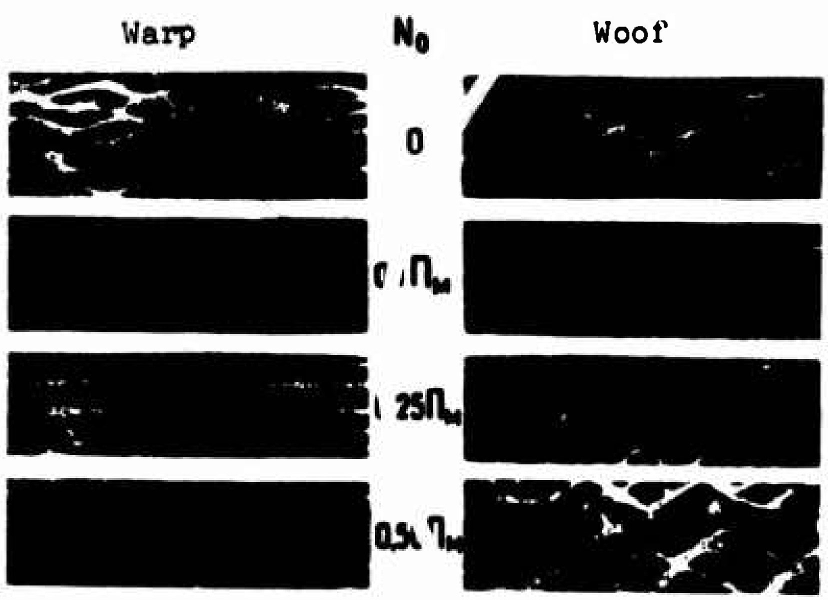


Fig. 1.4.8. Macrostructure of woven materials (mag. 10×).

The tension of fibers for laminate materials of various structure substantially increases the modulus of elasticity $E_x^{+(-)}$; however, the straightening of longitudinal fibers of woven glass-fiber reinforced plastics is accompanied by additional bending of the transverse fibers (see Fig. 1.4.8). Consequently, the increase in modulus of elasticity on the warp because of the preliminary tension of the longitudinal reinforcement leads to a decrease in the modulus of elasticity on the woof. This makes inefficient (as indicates numerical calculation of the bend of plates from woven materials) the application of production-process tension only along the warp (or woof) during the preparation of flat parts from laminate materials. Simultaneous tension on reinforcement in the direction of the warp and woof leads to a considerable increase in the moduli of elasticity in both directions (Table 1.4.3); the amplitudes of distortion of the warp and woof sharply decrease [101]. If tension is applied only in one direction, then the Poisson coefficients in the plane of the plate also depend on the tension (Table 1.4.3). With simultaneous identical tension along the warp and woof ν_{xy} and ν_{yx} remain unchanged.

For woven materials of laminate structure in the examined range of tension it was not possible to obtain a break in the relationship $E_{x(y)}=f(N_{э\Phi})$;[1] however, a further increase in tension $N_{э\Phi}>0.5\,\Pi_M$ should not lead to a growth of $E_x^{+(-)}$. At $N_{э\Phi}=0.5\,\Pi_M$ the fibers of the examined material are already practically straight (Fig. 1.4.8c). Moreover, as a result of local breaks of the reinforcing fibers at $N_{э\Phi}>0.5\,\Pi_M$ the modulus of elasticity should drop. For glass-fiber reinforced plastics reinforced by linen weave fabric of uniform strength the most effective preliminary tension in one direction is the force $N_x=0.25\,\Pi_M$, at which is the greatest "total" rigidity is obtained. In three-dimensional cross-linked materials the modulus of elasticity in the direction of strands of the warp

[1]O. G. Tsyplakov [258] based on experiment recommends for large-scale articles T_0 = (0.1=0.2) Π_M, while for woven reinforcement smaller tensions erroneously are recommended, and for unidirectional greater tensions.

increases only in the initial section of the relationship $E_{x(x)}=f(N_{э\phi})$. For the examined materials $N_{э\phi} \approx 0.25\,\Pi_{M}$. A further increase in tension does not lead to a growth of E_x, inasmuch as the fibers of the woof of these materials form a three-dimensional frame which impedes the further straightening of the twisting of fibers of the warp.

It is necessary to note that the tension of reinforcement practically does not change the form of $\sigma \sim \varepsilon$ curves for the examined materials. In the examined range of change in stresses the acting stress does not prove to have an apparent effect on the modulus of elasticity $E_x^{+(-)}$, i.e., during elongation there is no further apparent straightening of strands, but during contraction the actual stresses prove to be considerably lower than the minimum critical stress s^* at which it is possible to expect local loss of stability of the reinforcing fibers. Deformation diagrams indicate also that the stress $\hat{\sigma}$ [263] at which a break in the relationship $\sigma \sim \varepsilon$ is observed somewhat shifts toward an increase with an increase in tension. All experiments on the effect of tension on deformation strands of a material have been made up to stress $\hat{\sigma}$.

In accordance with the theory given in the previous section, the tension of fibers should not affect the modulus of interlamination shear. Experimental data obtained during tests of unidirectional materials are presented in Table 1.4.2. Their statistical treatment indicates that G_{xz} is practically independent of tension. Tests on laminate samples reinforced by fabric[1] (see [102]) allow the same conclusions. In samples with three-dimensional cross-linked arrangement G_{xz} should depend (at least in the initial section $G_{xz(x)}=f(N_{э\phi})$) upon the tension of substantially twisted fibers of the warp. However, this question requires additional study.

---

[1]Experimental research on the effect of tension on the modulus of interlamination shear G_{xz} was conducted indirectly - bending samples with different ratio height to span. It is necessary to note the large variance of results during the testing of samples reinforced by fabric, explained by the fact that the accuracy of the accepted procedure is greater the stronger the effect of transverse force on the bending deflection (determined by the degree of anisotropy).

The data about the effect of distortion and additional tension of fibers on the parameter of anisotropy of three examined types of materials are collected in Table 1.4.4. They experimentally confirm earlier conclusion about the greater sensitivity to shear stresses of materials with straightened fibers as a result of growth of parameter β with the straightening of the reinforcing fibers.

Table 1.4.4. Effect of the tension of fibers on the degree of anisotropy $\beta=\sqrt{\frac{E_x}{G_{xz}}}$ of materials with the different structure of reinforcement.

Material	Twisted fibers	Straightened unstressed fibers	Preliminary tension of reinforcement, in fractions of Π_M		
			0,1	0,25	0,50
АГ-4С	4,38	5,05	5,31	5,26	5,24
СКТ-11	—	3,98	—	4,25	4,5
Three-dimensional cross-linked	2,94	2,90	3,15	—	—

The widespread delusion about the considerable distinction of moduli of elasticity E_x+ and E_x- y in composite materials reinforced by unstressed fibers (for example, in oriented glass-fiber reinforced plastics with fibrous or laminate structures) is the consequence of an incorrect choice of shape and dimensions of samples for contraction testing. The effect of straightening and stretching fibers on the various-resistivity of materials of different structure is shown in Fig. 1.4.9 and in special separate lines of Tables 1.4.2 and 1.4.3 (data were obtained during tensile and contraction tests of the same samples). It has been established that for a number of materials - unidirectional and woven, reinforced by straightened but unstressed fibers - the ratio $\frac{E_x+}{E_x-}$, characterizing modulus difference, is near one. The production-process stretching of reinforcement does not increase the various-resistivity materials with a fibrous structure. For them the ratio to the moduli of elasticity in

extension-contraction practically does not change when the reinforcement is stretched. In laminate woven materials reinforced by cellular fibers various-resistivity during extension-contraction has been found, while the ratio $\frac{E_x^+}{E_x^-}$ increases with an increase in the tension of the fibers, and the degree of modulus difference can be very considerable. This can lead to the necessity of reviewing the methods of designing parts from materials with cellular fibers. The tension of the reinforcement changes basically the elastic characteristics in the direction of the fibers. This facilitates the creation of a theory of various-resistivity anisotropic materials. For example, for problems of the curvature of rods it is necessary to take into account only the difference in E_x^+, E_x^-.

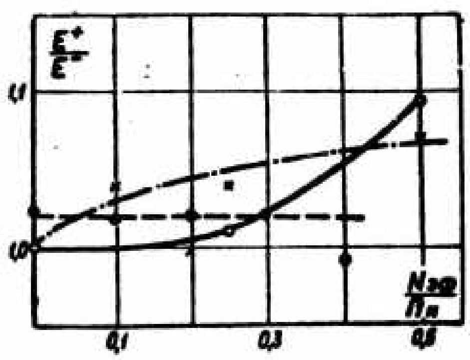


Fig. 1.4.9. The effect of the tension of fibers on the various-resistivity to extension and contraction in the direction of reinforcement. ● - AG-4S; × - SKT-11 (250°C); o - SKT-11 (20°C). In parentheses is the temperature of heated treatment of the reinforcing fabric.

1.4.4. *The effect of stretching on strength.* The effect of production-process stretching on strength is exhibited not only in the straightening of the fibers, but also in the generation of a system of residual stresses, which, depending on the acting load, can increase or decrease the strength of the material. For a qualitative appraisal of the role of tension we will use the data given in [102].[2] The data were obtained during tensile tests,

[1]The theory of elasticity of various-modulus materials is being developed by S. A. Ambartsumyan and his colleagues (see, for example, [12, 16, 18]).

[2]The effect of tension on the strength of reinforced plastics and the choice of the amount of this force have been incompletely studied. Some data are contained in [279, 295].

contraction tests and tests on the shear of plane and circular samples with different macrostructure of reinforcement. In Table 1.4.5 are compared the data obtained during tests of strip samples from AG-4S with straightened, but unstressed and preliminarily stressed fibers. They indicate a small effect of tension on strength during extension, and at the same time indicate a substantial increase in the stability of results when the reinforcements are stretched. This effect is insignificant only in comparison with a material for which the reinforcing fibers have been straightened. When there has been production process twisting, the strength of unidirectional AG-4S material is considerably below the data. In the parts made from material of this brand by the usual technology used in plants Π_x^+ constitutes 28-35 kgf/mm^2 [235]. Even more considerable is the drop in strength of unidirectional materials on polyester binders (much contraction), when the hardening took place in the absence of tension on the fibers [293].

Table 1.4.5. The effect of tension on the strength of unidirectional materials (AG-4S).

Strength,* kgf/cm^2	Usual technology of preparation [235]	Straightened unstressed fibers	Preliminarily stressed fibers: Force of tension, in fractions of $\Pi_{\text{м}}$			
			0,1	0,2	0,3	0,4
Π_x^+	28—35	69,0	72,0	71,5	71,0	75,9
$\frac{\Pi_x^{н}}{\Pi_x^{0}}$	0,40—0,50	1,00	1,04	1,03	1,03	1,09
Π_{xz}	—	3,60	4,37	4,39	4,15	4,10
$\frac{\Pi_{xz}^{н}}{\Pi_{xz}^{0}}$	—	1,00	1,22	1,22	1,16	1,14
$\frac{\Pi_x^{н}}{\Pi_{xz}^{н}}$	—	19,3	16,5	16,3	17,0	18,5

*The contraction of unidirectional materials could not be correctly tested. Plane samples crumpled at the loading place, moreover, the working stresses appearing in this case corresponded to the tabulated data about the strength of materials of an indicated type during contraction.

Table 1.4.5. contains data about shear strength, obtained during bending tests. Weak shear resistance somewhat decreases as a result of the production-process stretching of the reinforcement. However, the preliminary stretching of the reinforcement does not remove this negative characteristic of unidirectional materials.

Analogous results were obtained during tests of rings from unidirectional glass-fiber reinforced plastics on different binders [61, 99]. It has been shown that to achieve maximum strength it is sufficient to have a low tension on the mounting. Strength during extension increases with the tension on the fibers only to (0.05-0.10) Π_M. The necessary amounts of production-process for unidirectional materials are determined mainly by the contraction of the binder. In the examined range of change $N_{\text{ж}\Phi}$ = (0.05-0.80) Π_M the lack of a connection between tension and strength is apparent. The nature of the failure during the extension of unstressed and stressed samples from materials of fibrous structure, the test of which was made under the same conditions, indicates that preliminary tension leads to more uniform loading of the fibers.

The data about the strength of woven materials of laminate structure are presented in Fig. 1.4.11. They were obtained during tests of plane and circular samples from glass fiber laminates of various interweaving with tension along the warp or with simultaneous tension in the warp and woof. Samples cut out along the warp and woof were examined. It was established that strength during extension and contraction in the direction of the tension substantially depends on the amount of this force. For example, for material VPS-7 (Fig. 1.4.10) $\Pi_{\bullet}$ increases in proportion to the force of tension in the whole examined range of change $N_{\bullet\Phi}$.[1] Of all examined quantities of SKT-11 contraction strength increased the most (by almost twice). The growth of strength during contraction is explained

[1]In studying the strength of laminate materials of all types, as during the study of rigidity, in the examined working range of change in tension an optimum amount of this force has not been found.

by the fact that the reinforcements are not only straightened - the effect of tension during extension - but also by the fact that in the material is created a favorable curve of residual stresses. This indicates the advisability of a stressed reinforcement of constructions from glass-fiber reinforced plastics which operate in the stability mode.

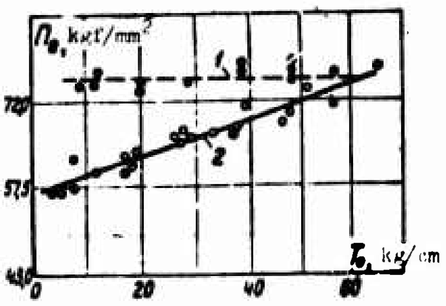


Fig. 1.4.10. The effect of tension of strip T_0 on the strength of circular samples Π_θ from unidirectional and woven materials: 1 - material LSB-F (diameter 120 mm, width 20 mm, depth 2.4 mm); 2 - material VPS-7 (diameter 200 mm, width 20 mm).

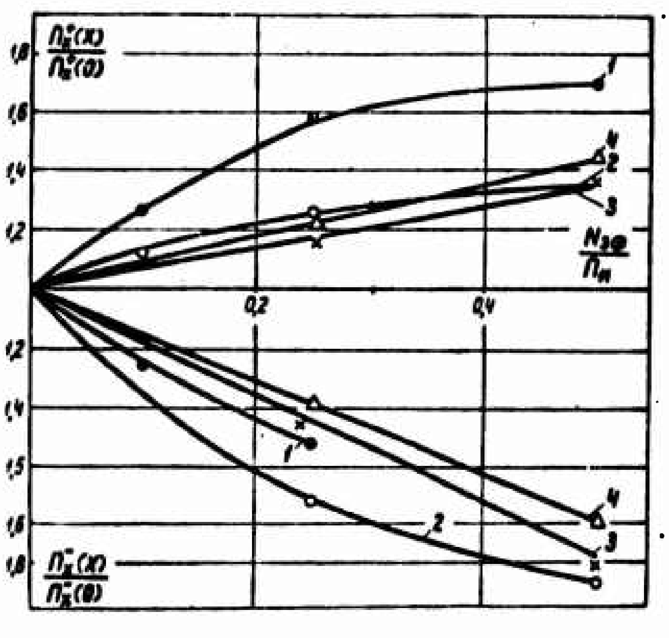


Fig. 1.4.11. The effect of tension on the strength of reinforced plastics with laminate structure: 1 - VPS-7; 2 - SKT-11 (250°C); 3 - SKT-11 (20°C); 4 - SKT-11 (400°C). In parentheses is the temperature of heat treatment of the reinforcing fabric; ●, O, x, Δ - experimental points.

Tension on the warp leads to a drop of strength along the wocf (Table 1.4.6). Tension in two mutually perpendicular directions removes this defect.

Table 1.4.6. The effect of tension* on the strength of woven materials.

Strength, kgf/cm^2	Laminate material (SKT-11)											Three-dimensional cross-linked				
	$\Pi_{x(x,0)}$				$\Pi_{y(x,0)}$				$\Pi_{x(x,y)}$			$\Pi_{x(x,0)}$			$\Pi_{x(x,y)}$	
	0	$0.10\Pi_M$	$0.25\Pi_M$	$0.50\Pi_M$	0	$0.10\Pi_M$	$0.25\Pi_M$	$0.50\Pi_M$	0	$0.10\Pi_M$	$0.25\Pi_M$	0	$0.10\Pi_M$	$0.25\Pi_M$	0	$0.15\Pi_M$
Π^+	13,0	14,2	16,0	17,0	11,0	7,9	7,5	6,8	13,0	14,8	17,0	35,9	44,5	57,0	35,9	44,5
$\frac{\Pi_M^+}{\Pi_0^+}$	1,00	1,09	1,27	1,34	1,00	0,72	0,68	0,62	1,00	1,14	1,31	1,0	1,24	1,59	1,0	1,24
Π^-	9,8	1,2	18,0	18,5	9,8	8,9	8,3	7,9	9,8	10,6	11,6	21,2	26,5	32,0	21,2	25,4
$\frac{\Pi_M^-}{\Pi_0^-}$	1,00	1,13	1,83	1,91	1,00	0,91	0,845	0,806	1,00	1,08	1,18	1,0	1,25	1,51	1,0	1,20

*Tension is given in fractions of Π_M.